

A logical framework for incremental type-checking

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PPS – Groupe de travail théorie des types et réalisabilité

A paradoxical situation

Observation

We have powerful tools to mechanize the metatheory of (proof) languages

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... And yet,

Workflow of programming and formal mathematics is still largely inspired by legacy software development (`emacs`, `make`, `svn`, `diffs`...)

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We have powerful tools to mechanize the metatheory of (proof) languages

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Workflow of programming and formal mathematics is still largely inspired by legacy software development (`emacs`, `make`, `svn`, `diffs`...)

Isn't it time to make these tools metatheory-aware?

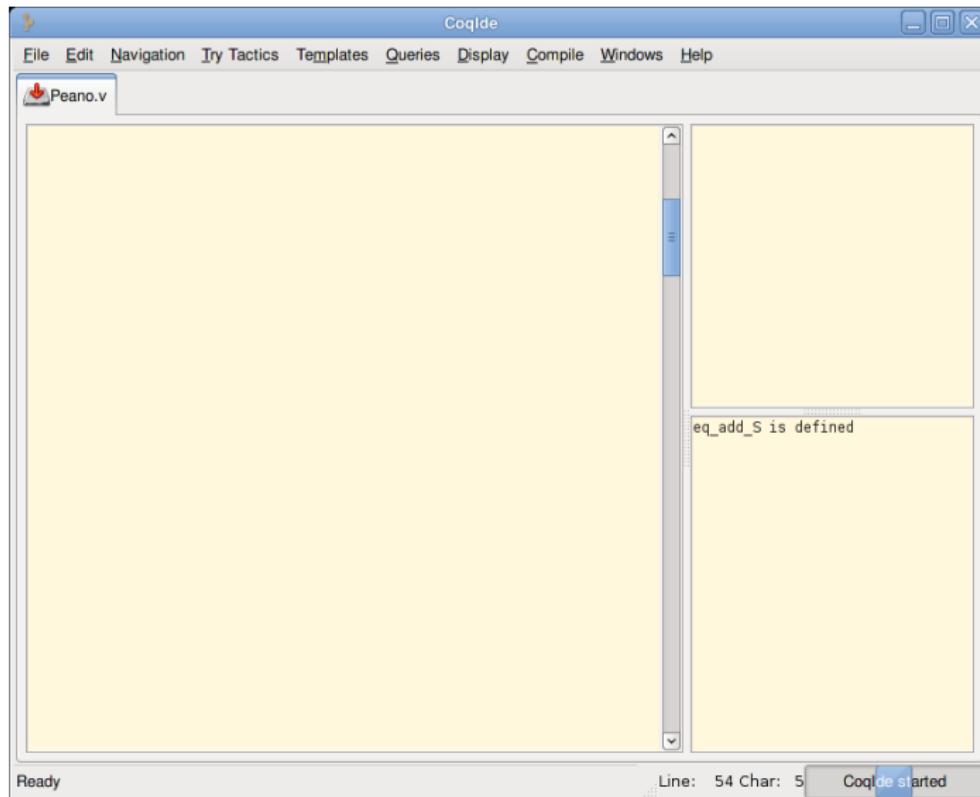
Incrementality in programming & proof languages

Q : Do you spend more time *writing* code or *editing* code?

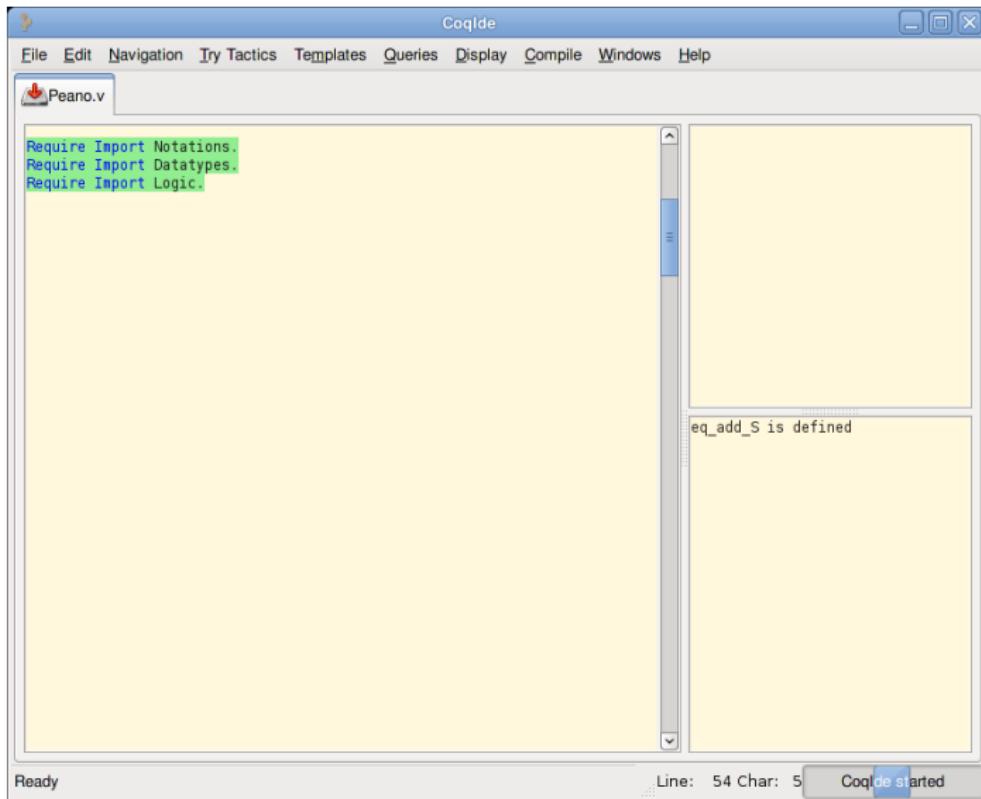
Today, we use:

- ▶ separate compilation
- ▶ dependency management
- ▶ version control on the scripts
- ▶ interactive toplevel with rollback (**Coq**)

Incrementality in programming & proof languages



Incrementality in programming & proof languages



Incrementality in programming & proof languages

```
Require Import Notations.
Require Import Datatypes.
Require Import Logic.

(** The predecessor function *)

Definition pred (n:nat) : nat := match n with
| 0 => n
| S u => u
end.

Theorem pred_Sn : forall n:nat, n = pred (S n).
Proof.
simpl; reflexivity.
Qed.

Theorem not_eq_S : forall n m:nat, n <> m -> S n <> S m.
Proof.
red in |- *; auto.
Qed.

(** Injectivity of successor *)

Theorem eq_add_S : forall n m:nat, S n = S m -> n = m.
Proof.
intros n m Sn_eq_Sm.
replace (n=m) with (pred (S n) = pred (S m)) by auto using pred_Sn.
rewrite Sn_eq_Sm; trivial.
Qed.

Definition IsSucc (n:nat) : Prop :=
match n with
| 0 => False
| S o => True
```

Ready Line: 54 Char: 5 CoqIDE started

Incrementality in programming & proof languages

The screenshot shows the CoqIDE interface with a file named "Peano.v". The code defines the predecessor function and proves its reflexivity. It then defines a theorem about the successor function being injective and proves it using the predecessor function's reflexivity. Finally, it defines the IsSucc predicate.

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Incrementality in programming & proof languages

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The status bar at the bottom indicates "Ready", "Line: 36 Char: 1", and "CoqIDE started".

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Proof.
  simpl; reflexivity. (* simple proof *)
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The status bar at the bottom indicates "Ready", "Line: 39 Char: 41", and "CoqIDE started".

Incrementality in programming & proof languages

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Theorem not_eq_S : forall n m:nat, n <= m -> S n <= S m.
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Incrementality in programming & proof languages

The screenshot shows the CoqIDE interface with a file named "Peano.v" open. The code implements basic arithmetic operations using the Peano axioms.

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The code includes several theorems and their proofs. The first theorem, `pred_Sn`, proves that the predecessor of a natural number is the natural number itself. The second theorem, `not_eq_S`, proves that if one natural number is less than or equal to another, then their successors are also less than or equal. The third part of the code defines the injectivity of the successor function, showing that if two numbers have the same successor, they must be equal. The final part defines a predicate `IsSucc` which returns `False` for 0 and `True` for any other natural number.

In an ideal world...

- ▶ Edition should be possible anywhere
- ▶ The impact of changes visible “in real time”
- ▶ No need for separate compilation, dependency management

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Types are good witnesses of this impact

Applications

- ▶ non-linear user interaction
- ▶ tactic languages
- ▶ type-directed programming
- ▶ typed version control systems

Menu

The big picture

Our approach

Why not memoization?

A popular storage model for repositories

Logical framework

Positionality

The language

From LF to NLF

NLF: Syntax, typing, reduction

Architecture

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A logical framework for incremental type-checking

Yes, we're speaking about (any) typed language.

A type-checker

val check : env → term → types → bool

- ▶ builds and checks the derivation (on the stack)
- ▶ conscientiously discards it

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A logical framework for incremental type-checking

Yes, we're speaking about (any) typed language.

A type-checker

val check : env → term → types → bool

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true

A logical framework for **incremental** type-checking

Goal Type-check a large derivation taking advantage of the knowledge from type-checking previous versions

Idea Remember all derivations!

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Q Do we really need faster type-checkers?

A Yes, since we implemented these ad-hoc fixes.

The big picture

version management

script files

parsing

type-checking

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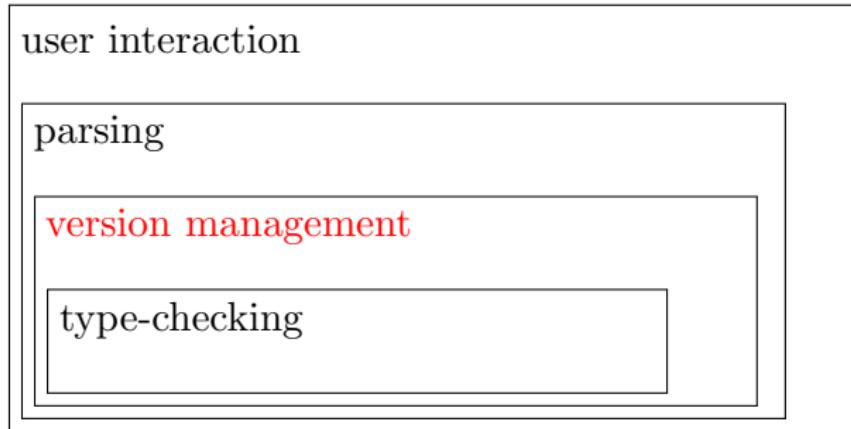
- ▶ AST representation

The big picture



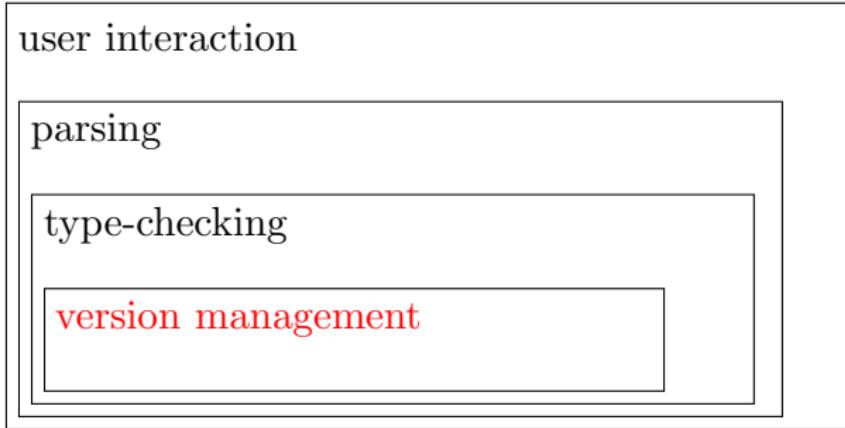
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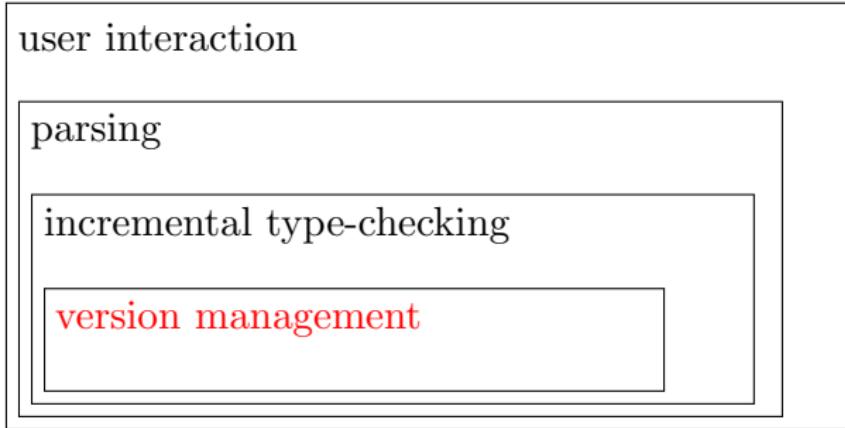
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The big picture



- ▶ AST representation
- ▶ Typing annotations

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Our approach

Why not memoization?

A popular storage model for repositories

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From LF to NLF

NLF: Syntax, typing, reduction

Architecture

Memoization maybe?

```
let rec check env t a =  
  match t with  
  | ... → ... false  
  | ... → ... true
```

```
and infer env t =  
  match t with  
  | ... → ... None  
  | ... → ... Some a
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Memoization maybe?

```
let table = ref ([] : environ × term × types) in
let rec check env t a =
  if List.mem (env,t,a) !table then true else
    match t with
    | ... → ... false
    | ... → ... table := (env,t,a)::! table ; true
and infer env t =
  try List.assoc (env,t) !table with Not_found →
    match t with
    | ... → ... None
    | ... → ... table := (env,t,a)::! table ; Some a
```

Memoization maybe?

- + lightweight

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What does it mean logically?

$$\frac{J \in \Gamma}{\Gamma \vdash J \text{ wf} \Rightarrow \Gamma}$$

$$\frac{\Gamma_1 \vdash J_1 \text{ wf} \Rightarrow \Gamma_2 \quad \dots \quad \Gamma_{n-1}[J_{n-1}] \vdash J_n \text{ wf} \Rightarrow \Gamma_n}{\Gamma_1 \vdash J \text{ wf} \Rightarrow \Gamma_n[J_n][J]}$$

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What if I want *e.g.* the weakening property to be taken into account?

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- still no trace of the derivation

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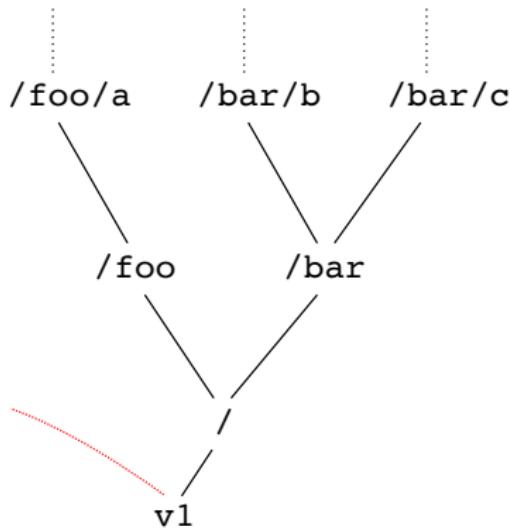
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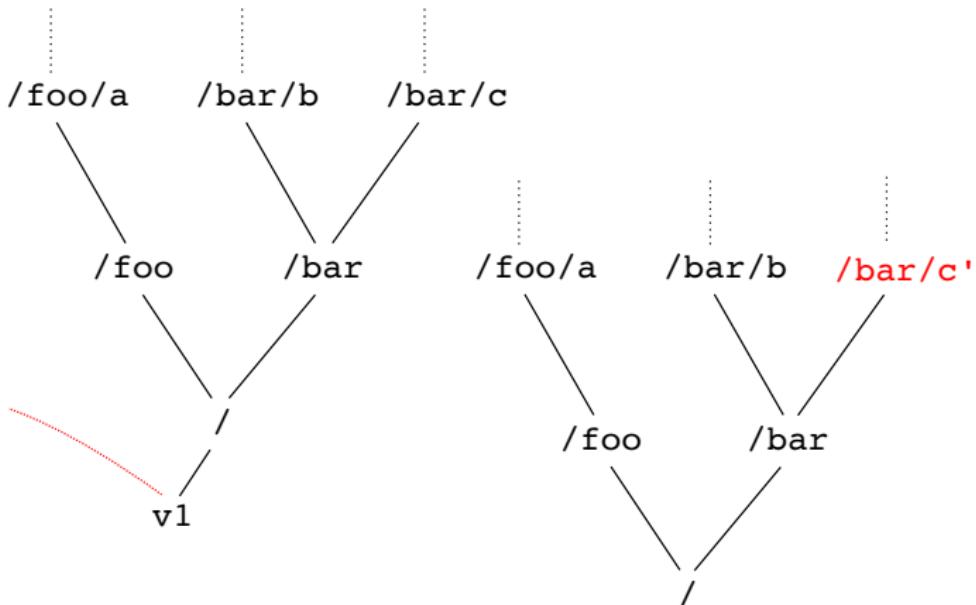
What if I want *e.g.* the weakening property to be taken into account?

- syntactic comparison
- still no trace of the derivation
- + gives good reasons to go on

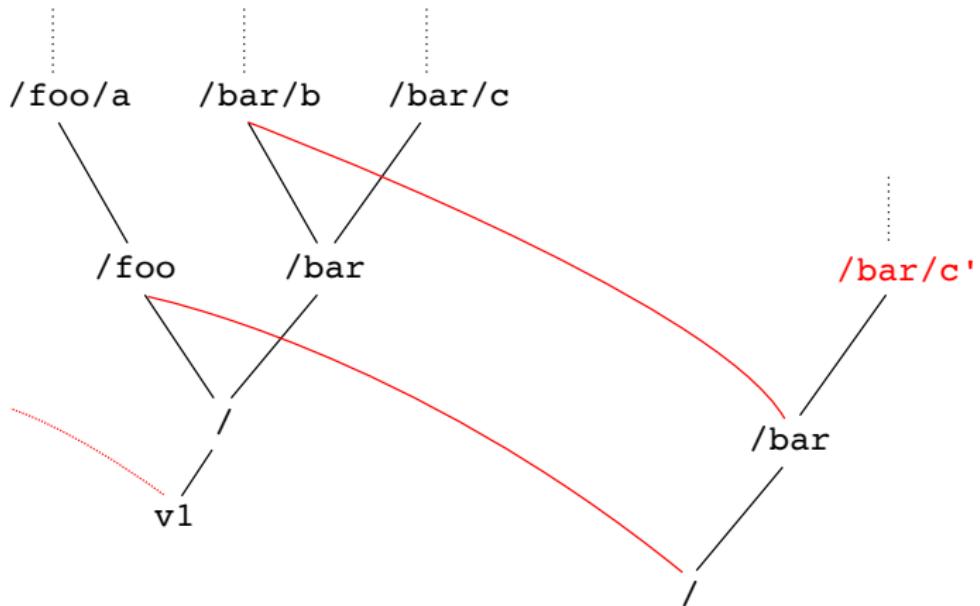
A popular storage model for repositories



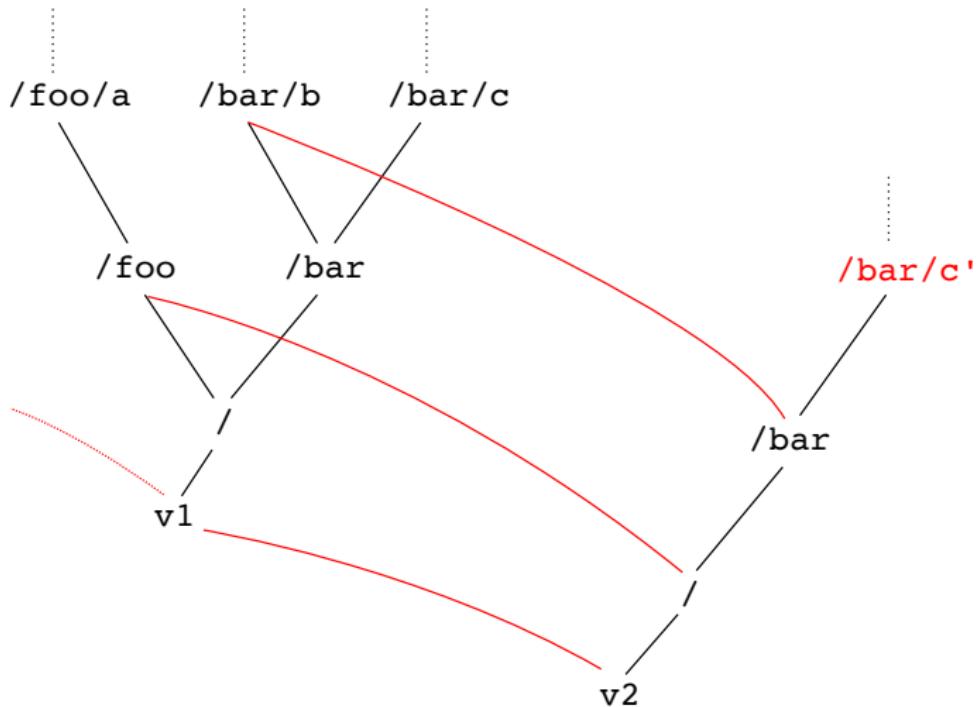
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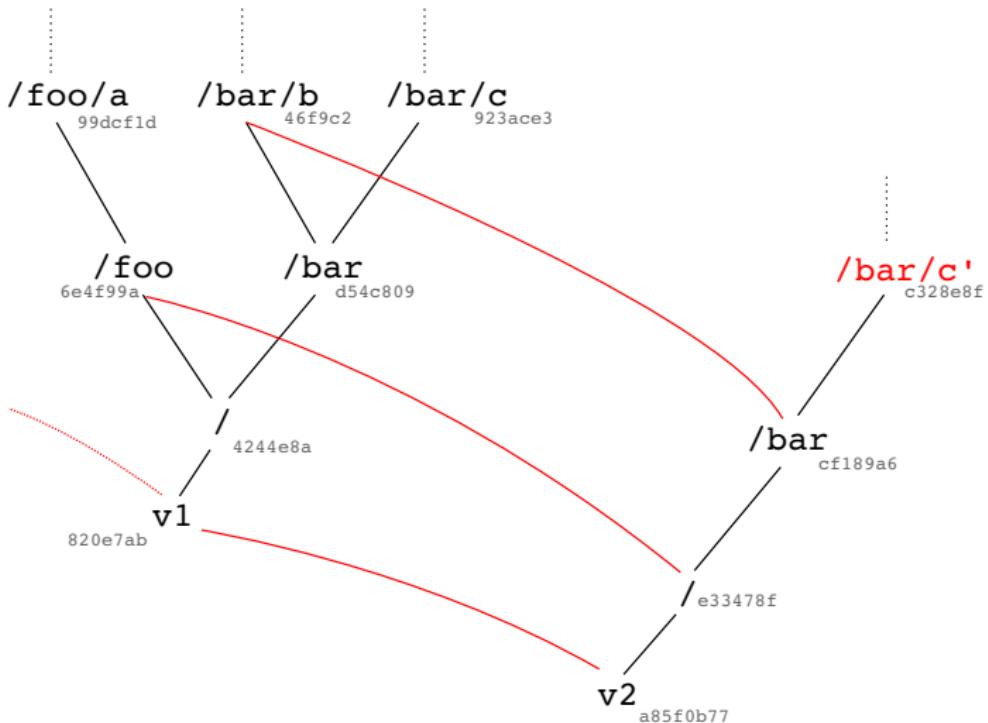
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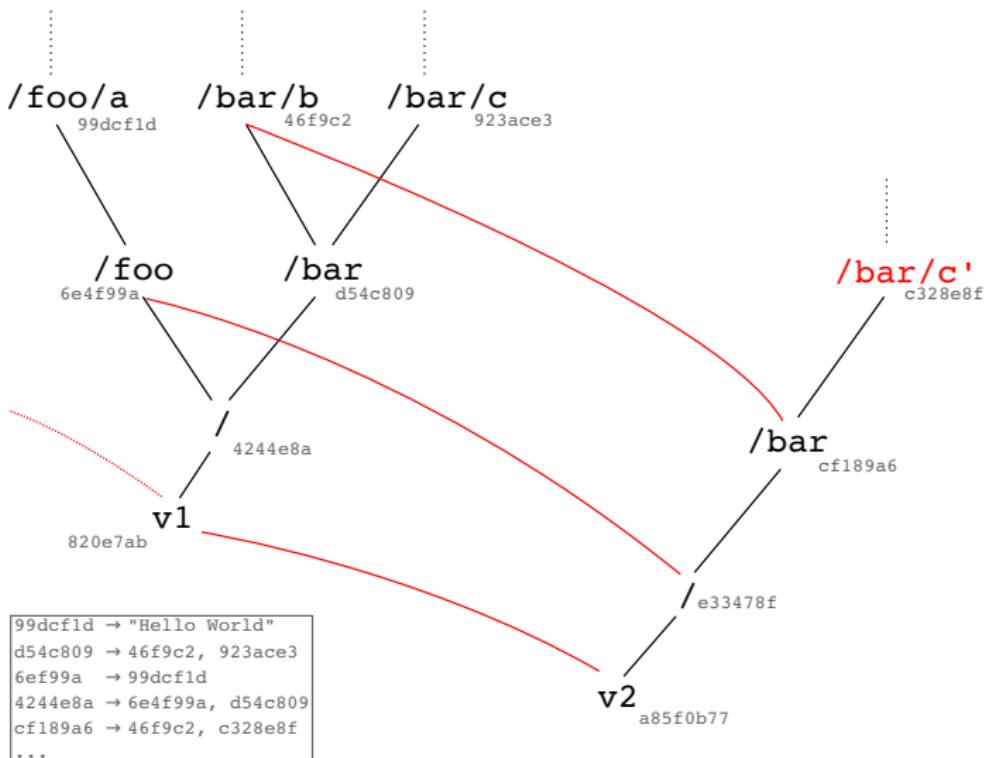
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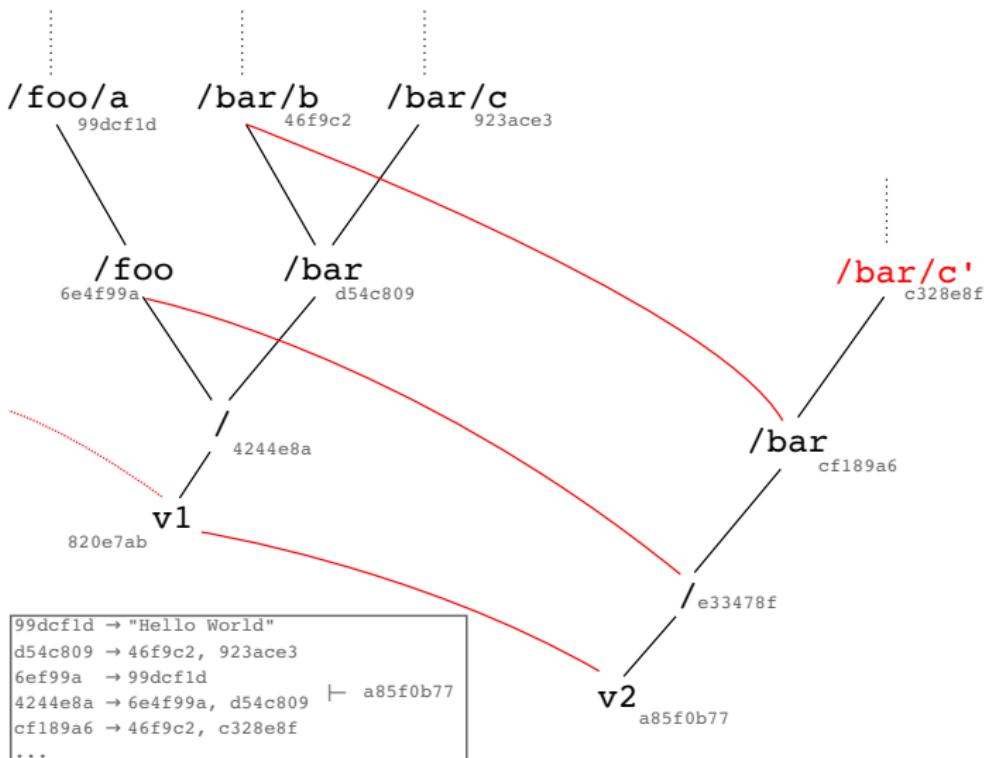
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A popular storage model for repositories

The repository R is a pair (Δ, x) :

$$\Delta : x \mapsto (\text{Commit } (x \times y) \mid \text{Tree } \vec{x} \mid \text{Blob } string)$$

with the invariants:

- ▶ if $(x, \text{Commit } (y, z)) \in \Delta$ then
 - ▶ $(y, \text{Tree } t) \in \Delta$
 - ▶ $(z, \text{Commit } (t, v)) \in \Delta$
- ▶ if $(x, \text{Tree}(\vec{y})) \in \Delta$ then
 - for all y_i , either $(y_i, \text{Tree}(\vec{z}))$ or $(y_i, \text{Blob}(s)) \in \Delta$

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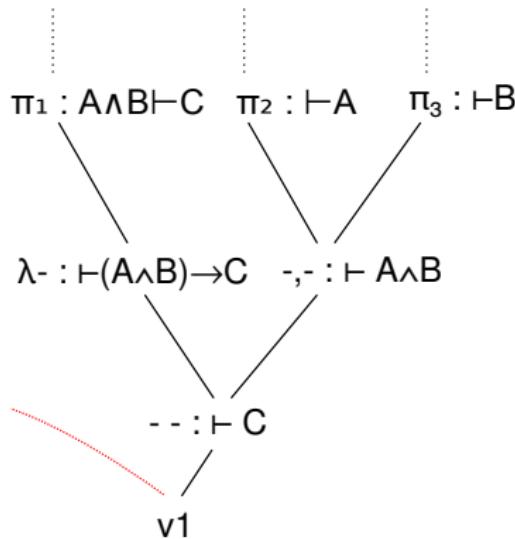
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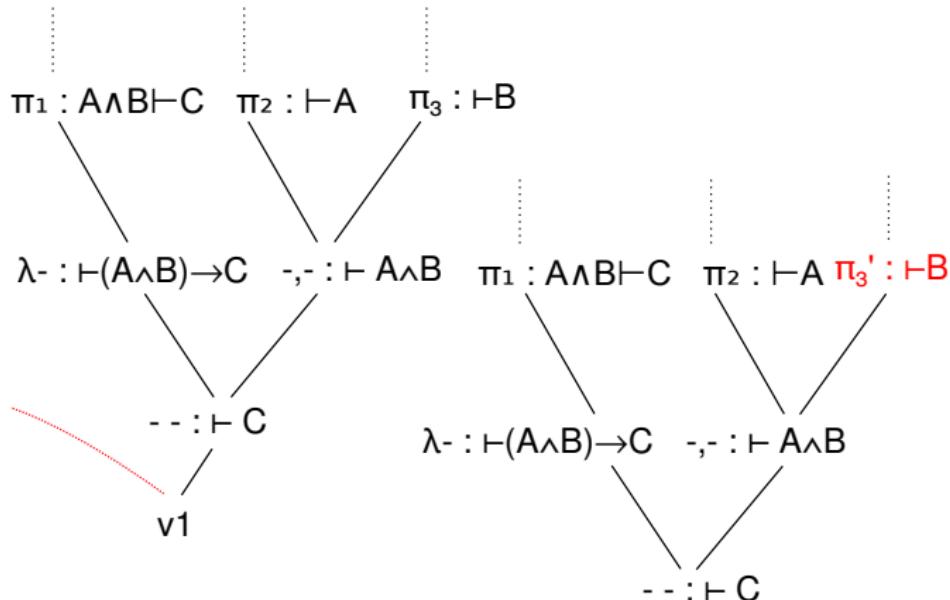
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Let's do the same with *proofs*

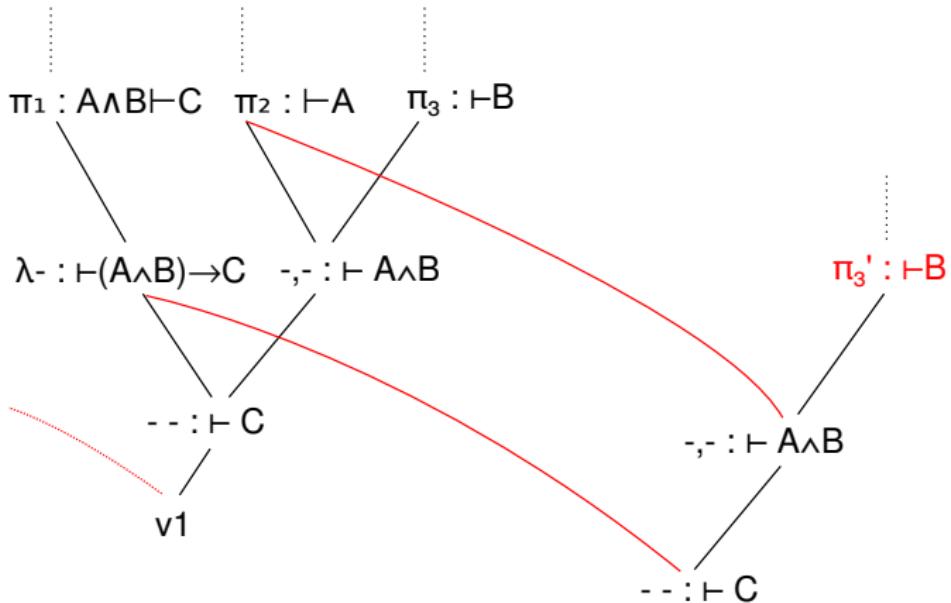
A *typed* repository of proofs



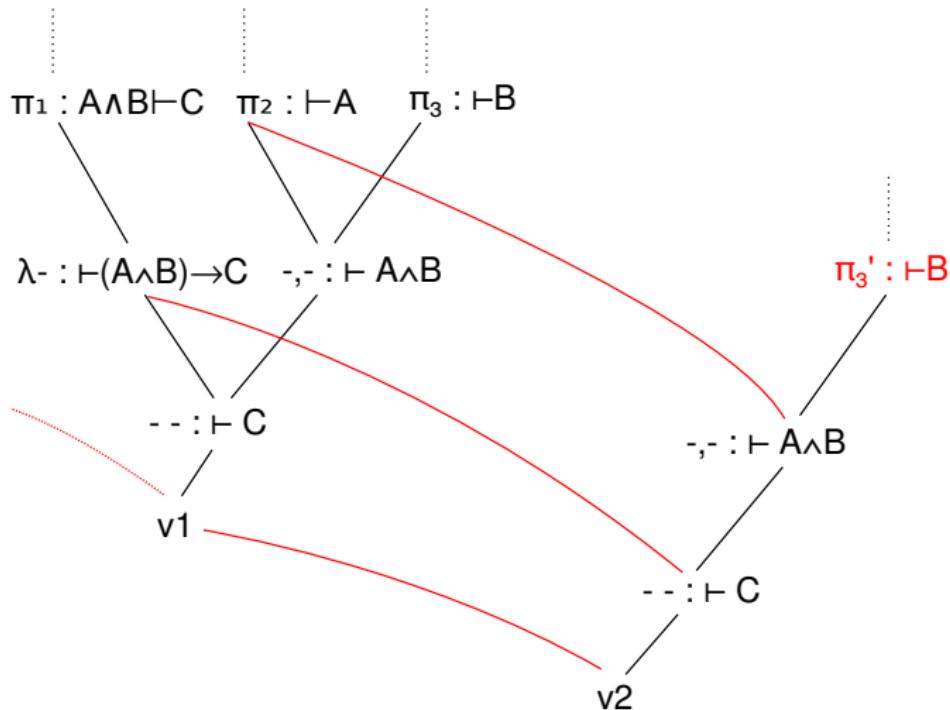
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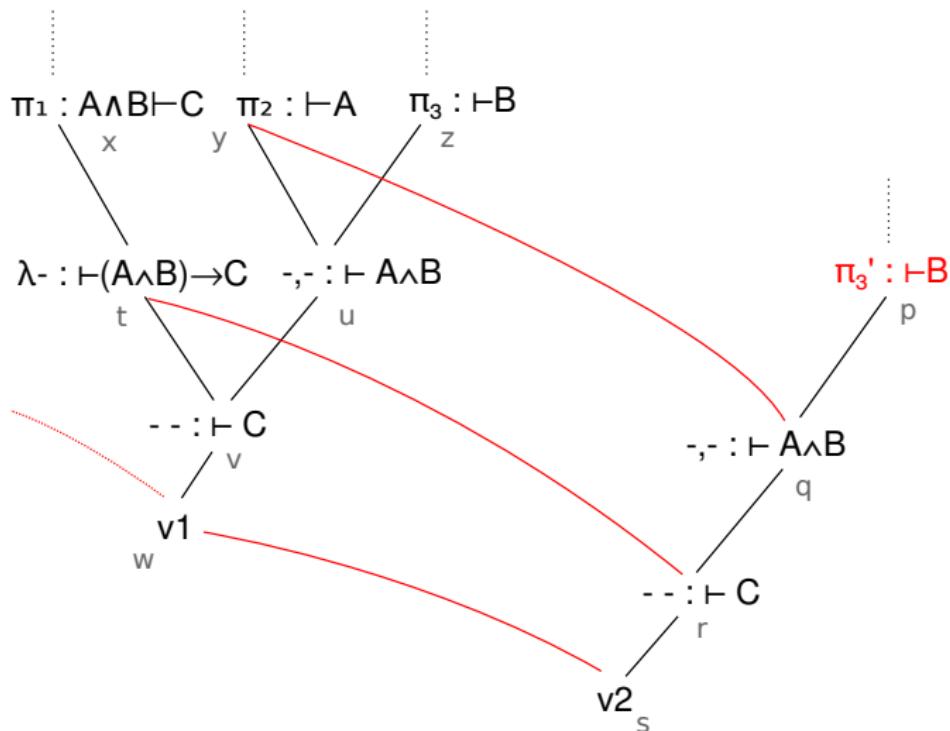
A typed repository of proofs



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A typed repository of proofs



A *typed* repository of proofs

$x = \dots : A \wedge B \vdash C$

$y = \dots : \vdash A$

$z = \dots : \vdash B$

$t = \lambda a^{A \wedge B} \cdot x : \vdash A \wedge B \rightarrow C$

$u = (y, z) : \vdash A \wedge B$

$v = t \ u : \vdash C$

$w = \text{Commit}(v, w1) : \text{Version}$

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A *typed* repository of proofs

```
let x = ... : is (cons (conj A B) nil) C in
  let y = ... : is nil A in
    let z = ... : is nil B in
      let t = lam (conj A B) x : is nil (arr (conj A B) C) in
        let u = pair y z : is nil (conj A B) in
          let v = app t u : is nil C in
            let w = commit v w1 : version in
              w
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              let p = ... : is nil B
                let q = pair y p : is nil (conj A B) in
                  let r = t q : is nil C
                    let s = commit r w : version in
                      s
```

A *typed* repository of proofs

```
...
val is : env → prop → type
val conj : prop → prop → prop
val pair : is α β → is α γ → is α (conj β γ)
val version : type
val commit : is nil C → version → version
...
```

```
let x = ... : is (cons (conj A B) nil) C in
  let y = ... : is nil A in
    let z = ... : is nil B in
      let t = lam (conj A B) x : is nil (arr (conj A B) C) in
        let u = pair y z : is nil (conj A B) in
          let v = app t u : is nil C in
            let w = commit v w1 : version in
              let p = ... : is nil B
                let q = pair y p : is nil (conj A B) in
                  let r = t q : is nil C
                    let s = commit r w : version in
                      s
```

A logical framework for incremental type-checking

LF [Harper et al. 1992] provides a way to represent and validate syntax, rules and proofs by means of a typed λ -calculus. But we need a little bit more:

```
...
let u = pair y z : is nil (conj A B) in
  let v = app t u : is nil C in
...
...
```

A logical framework for incremental type-checking

LF [Harper et al. 1992] provides a way to represent and validate syntax, rules and proofs by means of a typed λ -calculus. But we need a little bit more:

...
let $u = \text{pair } y \ z : \text{is nil } (\text{conj } A \ B)$ **in**
let $v = \text{app } t \ u : \text{is nil } C$ **in**

...
1. definitions / explicit substitutions

A logical framework for incremental type-checking

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...
```

1. definitions / explicit substitutions
2. type annotations on application spines

A logical framework for incremental type-checking

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let u = pair y z : is nil (conj A B) in
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1. definitions / explicit substitutions
2. type annotations on application spines
3. fully applied constants / η -long NF

A logical framework for incremental type-checking

LF [Harper et al. 1992] provides a way to represent and validate syntax, rules and proofs by means of a typed λ -calculus. But we need a little bit more:

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let u = pair y z : is nil (conj A B) in
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```

- 1. definitions / explicit substitutions
- 2. type annotations on application spines
- 3. fully applied constants / η -long NF
- 4. Naming of all application spines / A-normal form
(= construction of syntax/proofs)

Positionality

$R =$

```
let x = ... : is (cons (conj A B) nil) C in
  let y = ... : is nil A in
    let z = ... : is nil B in
      let t = lam (conj A B) x : is nil (arr (conj A B) C) in
        let u = pair y z : is nil (conj A B) in
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            let w = commit v w1 : version in
              w
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- ▶ Expose the *head* of the term

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- ▶ Expose the *head* of the term

$$(\lambda x. \lambda y. T) U V$$

Positionality

$R =$

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- ▶ Expose the *head* of the term

$$(\lambda x. \lambda y. T) \ U \ V$$

- ▶ Abstract from the *positions* of the binders
(from inside and from outside)

Menu

The big picture

Our approach

Why not memoization?

A popular storage model for repositories

Logical framework

Positionality

The language

From LF to NLF

NLF: Syntax, typing, reduction

Architecture

Presentation (of the ongoing formalization)

- ▶ alternative syntax for LF
- ▶ a datastructure of LF derivations
- ▶ the repository storage model

Motto: *Take control of the environment*



From LF to XLF

$K ::= \Pi x^A . K \mid *$

$A ::= \Pi x^A . A \mid A\ t \mid a$

$t ::= \lambda x^A . t \mid \text{let } x = t \text{ in } t \mid t\ t \mid x \mid c$

$\Gamma ::= \cdot \mid \Gamma[x : A] \mid \Gamma[x = t]$

$\Sigma ::= \cdot \mid \Sigma[c : A] \mid \Sigma[a : K]$

- ▶ start from standard λ_{LF} with definitions

From LF to XLF

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- ▶ sequent calculus-like applications ($\bar{\lambda}$)
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$$\frac{\Gamma \vdash t : A \quad \Gamma, B\{x/t\} \vdash l : C}{\Gamma, \Pi x^A \cdot B \vdash t; l : C}$$

From LF to XLF

$K ::= \Pi x^A \cdot K \mid *$

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- ▶ start from standard λ_{LF} with definitions
- ▶ sequent calculus-like applications ($\bar{\lambda}$)
- ▶ type annotations on application spines

$$\frac{\Gamma \vdash t : A \quad \Gamma[x = t], \color{red}{B} \vdash l : C}{\Gamma, \Pi x^A \cdot B \vdash t; l : C}$$

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$\Gamma ::= \cdot \mid \Gamma [x : A] \mid \Gamma [x = t]$

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- ▶ start from standard λ_{LF} with definitions
- ▶ sequent calculus-like applications ($\bar{\lambda}$)
- ▶ type annotations on application spines
- ▶ named arguments

$$\frac{\Gamma \vdash t : A \quad \Gamma [x = t], B \vdash l : C}{\Gamma, \Pi x^A \cdot B \vdash \color{red}{x = t; l} : C}$$

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- ▶ start from standard λ_{LF} with definitions
- ▶ sequent calculus-like applications ($\bar{\lambda}$)
- ▶ type annotations on application spines
- ▶ named arguments

$$\mathsf{FV}(t[l] : A) = \mathsf{FV}(t) \cup \mathsf{FV}(l) \cup (\mathsf{FV}(A) - \mathsf{FV}(l))$$

$$\mathsf{FV}(x = t; l) = \mathsf{FV}(t) \cup (\mathsf{FV}(l) - \{x\})$$

XLF: Properties

- ▶ LJ-style application
- ▶ type annotation on application spines
- ▶ named arguments (labels)

Lemma (Conservativity)

- ▶ $\Gamma \vdash_{\text{LF}} K \text{ kind} \quad iff \quad |\Gamma| \vdash_{\text{XLF}} |K| \text{ kind}$
- ▶ $\Gamma \vdash_{\text{LF}} A \text{ type} \quad iff \quad |\Gamma| \vdash_{\text{XLF}} |A| \text{ type}$
- ▶ $\Gamma \vdash_{\text{LF}} t : A \quad iff \quad |\Gamma| \vdash_{\text{XLF}} |t| : |A|$

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- ▶ start from XLF

From XLF to NLF

$K ::= \Pi x^A \cdot K \mid h_K$

$h_K ::= *$

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- ▶ start from XLF
- ▶ isolate heads (non-binders)

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$h_t ::= t[l] : A \mid x[l] : A \mid c[l] : A$

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$h_A ::= A[l] : \textcolor{red}{h_K} \mid a[l] : \textcolor{red}{h_K}$

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- ▶ start from XLF
- ▶ isolate heads (non-binders)
- ▶ enforce η -long forms by annotating with heads

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$$t[l] : \Pi x^A \cdot B \longrightarrow_{\eta} \lambda x^A \cdot (t[x = x; l] : B) \quad x \notin \text{FV}(t)$$

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$h_A ::= A[l] : h_K \mid a[l] : h_K$

$t ::= \Gamma \vdash h_t$

$h_t ::= t[l] : h_A \mid x[l] : h_A \mid c[l] : h_A$

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- ▶ start from XLF
- ▶ isolate heads (non-binders)
- ▶ enforce η -long forms by annotating with heads
- ▶ factorize binders and environments

From XLF to NLF

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- ▶ start from XLF
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$h_t ::= t[l] : h_A \mid x[l] : h_A \mid c[l] : h_A$

$l ::= \Gamma$

$\Gamma : x \mapsto ([x : A] \mid [x = t])$

$\Sigma ::= \cdot \mid \Sigma [c : A] \mid \Sigma [a : K]$

- ▶ start from XLF
- ▶ isolate heads (non-binders)
- ▶ enforce η -long forms by annotating with heads
- ▶ factorize binders and environments
- ▶ abstract over environment datastructure (maps)

NLF

Syntax

$$K ::= \Gamma \text{ kind}$$
$$A ::= \Gamma \vdash h_A \text{ type}$$
$$h_A ::= a \ \Gamma$$
$$t ::= \Gamma \vdash h_t : h_A$$
$$h_t ::= t \ \Gamma \mid x \ \Gamma \mid c \ \Gamma$$
$$\Gamma : x \mapsto ([x : a] \mid [x = t])$$

Judgements

- ▶ $\Gamma \text{ kind}$
- ▶ $\Gamma \vdash h_A \text{ type}$
- ▶ $\Gamma \vdash h_t : h_A$

NLF

Syntax

$K ::= \Gamma \text{ kind}$

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$\Gamma ::= x \mapsto ([x : a] \mid [x = t])$

Judgements

► K

► A

► t

NLF

Syntax

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$h_t ::= t \ \Gamma \mid x \ \Gamma \mid c \ \Gamma$

$\Gamma : x \mapsto ([x : a] \mid [x = t])$

Judgements

- ▶ $K \text{ wf}$
- ▶ $A \text{ wf}$
- ▶ $t \text{ wf}$

NLF

Notations

- ▶ “ h_A ” for “ $\vdash h_A$ ”
- ▶ “ h_A ” for “ $\emptyset \vdash h_A$ type”
- ▶ “ a ” for “ $a \ \emptyset$ ”

Example

$$\lambda f^{A \rightarrow B} \cdot \lambda x^A \cdot f \ x : (A \rightarrow B) \rightarrow A \rightarrow B \quad \equiv \\ [f : [a : A] \vdash B \text{ type}] [x : A] \vdash f \ [a = x] : B$$

Some more examples

$A : \emptyset \text{ kind}$

$\equiv *$

$vec : [len : \mathbb{N}] \text{ kind}$

$\equiv \mathbb{N} \rightarrow *$

$nil : \vdash vec [len = \vdash 0 : \mathbb{N}] \text{ type}$

$\equiv vec 0 : *$

$cons : [l : \mathbb{N}] [hd : A] [tl : \vdash vec [len = \vdash l : \mathbb{N}] \text{ type}] \vdash$
 $vec [len = \vdash s [n = \vdash l : \mathbb{N}] : \mathbb{N}] \text{ type}$

$\equiv \Pi l^{\mathbb{N}} \cdot A \rightarrow \Pi tl^{vec\ l} \cdot vec (s\ l : \mathbb{N}) : *$

$fill : [n : \mathbb{N}] \vdash vec [len = \vdash n : \mathbb{N}] \text{ type}$

$\equiv \Pi n^{\mathbb{N}} \cdot (vec\ n : *)$

$empty : [e : vec [len = 0]] \text{ kind}$

$\equiv vec 0 \rightarrow *$

$- : \vdash empty [e = \vdash fill [n = 0] : vec [len = n]] \text{ type}$

$\equiv empty (fill 0 : vec 0)$

Environments

... double as labeled *directed acyclic graphs* of dependencies:

Definition (environment)

$\Gamma = (V, E)$ directed acyclic where:

- ▶ $V \subseteq \mathcal{X} \times (t \uplus A)$ and
- ▶ $(x, y) \in E$ (x depends on y) if $y \in \text{FV}(\Gamma(x))$

Definition (lookup)

$$\Gamma(x) : A \quad \text{if} \quad (x, A) \in E$$

$$\Gamma(x) = t \quad \text{if} \quad (x, t) \in E$$

Definition (bind)

$$\Gamma[x : A] = (V \cup (x, A), E \cup \{(x, y) \mid y \in \text{FV}(A)\})$$

$$\Gamma[x = t] = (V \cup (x, A), E \cup \{(x, y) \mid y \in \text{FV}(t)\})$$

Environments

... double as labeled *directed acyclic graphs* of dependencies:

Definition (decls, defs)

$\text{decls}(\Gamma) = [x_1, \dots, x_n]$ s.t. $\Gamma(x_i) : A_i$ topologically sorted wrt. Γ
 $\text{defs}(\Gamma) = [x_1, \dots, x_n]$ s.t. $\Gamma(x_i) = t_i$ topologically sorted wrt. Γ

Definition (merge)

$\Gamma \cdot \Delta = \Gamma \cup \Delta$ s.t.

- ▶ if $\Gamma(x) : A$ and $\Gamma(x) = t$ then $\Gamma \cdot \Delta(x) = t$
- ▶ undefined otherwise

Reduction

Definition (A-contexts)

$$\Delta^* = \{ [x = x] \mid x \in \text{defs}(\Delta) \}$$

$$\begin{array}{lll} \Gamma \vdash (\Delta \vdash h_t : h_A) \Xi : _ & \xrightarrow{\text{"}\beta\text{"}} & \Gamma \cdot \Delta \cdot \Xi \vdash h_t : h_A \\ \Gamma \vdash c \Delta : h_A & \longrightarrow & \Gamma \cdot \Delta \vdash c \Delta^* : h_A \quad \text{if } \Delta \neq \Delta^* \\ \Gamma \vdash c \Xi^* : a \Delta & \longrightarrow & \Gamma \cdot \Delta \vdash c \Xi^* : a \Delta^* \quad \text{if } \Delta \neq \Delta^* \end{array}$$

Typing

$$\frac{\text{FAM} \quad \Sigma(a) : (\Xi \text{ kind}) \quad \Gamma \vdash \Delta : \Xi}{\Gamma \vdash a \Delta \text{ type}}$$

$$\frac{\text{OBJC} \quad \Sigma(c) : (\Xi \vdash h_A \text{ type}) \quad \Gamma \vdash \Delta : \Xi \quad \Gamma \cdot \Xi \cdot \Delta \vdash h'_A \equiv h_A \text{ type}}{\Gamma \vdash c \Delta : h'_A}$$

$$\frac{\text{OBJX} \quad \Gamma(x) = (\Xi \vdash h_t : h_A) \quad \Gamma \vdash \Delta : \Xi \quad \Gamma \cdot \Xi \cdot \Delta \vdash h'_A \equiv h_A \text{ type} \\ \Gamma \cdot \Xi \cdot \Delta \vdash h_t : h_A}{\Gamma \vdash x \Delta : h'_A}$$

$$\frac{\text{ARGS } \forall x \in \mathbf{decls}(\Xi) \quad \Delta(x) = (\Delta' \vdash h_t : h_A) \quad \Xi(x) : (\Xi' \vdash h'_A \text{ type}) \\ \Gamma \cdot \Delta \cdot \Delta' \vdash h_t : h_A \quad \Gamma \cdot \Xi \cdot \Delta \cdot \Xi' \cdot \Delta' \vdash h'_A \equiv h_A \text{ type}}{\Gamma \vdash \Delta : \Xi}$$

Properties

Translation functions

- ▶ $|\cdot|_\Gamma : K_{\text{LF}} \rightarrow \Gamma_{\text{NLF}} \rightarrow K_{\text{NLF}}$ option
- ▶ $|\cdot|_\Gamma : A_{\text{LF}} \rightarrow \Gamma_{\text{NLF}} \rightarrow A_{\text{NLF}}$ option
- ▶ $|\cdot|_\Gamma : t_{\text{LF}} \rightarrow \Gamma_{\text{NLF}} \rightarrow t_{\text{NLF}}$ option
- ▶ ... and their inverses $|\cdot|^{-1}$

Conjecture (Conservativity)

- ▶ $\vdash_{\text{LF}} K \text{ kind} \quad \text{iff} \quad (|K|_\emptyset) \text{ wf}$
- ▶ $\vdash_{\text{LF}} A \text{ type} \quad \text{iff} \quad (|A|_\emptyset) \text{ wf}$
- ▶ $\vdash_{\text{LF}} t : A \quad \text{iff} \quad (|t|_\emptyset) \text{ wf}$

Menu

The big picture

Our approach

Why not memoization?

A popular storage model for repositories

Logical framework

Positionality

The language

From LF to NLF

NLF: Syntax, typing, reduction

Architecture

Status

```
$ ./gasp init hol.elf
```

Status

```
$ ./gasp init hol.elf  
  
[holtype : kind]  
[i : holtype]  
[o : holtype]  
[arr : [x2 : holtype][x1 : holtype] ⊢ holtype type]  
  
Fatal error: exception Assert_failure("src/NLF.ml", 61, 13)
```

Checkout

```
$ ./gasp checkout v42
```

if

$$t = \Gamma \vdash v_{52} : \text{Version} \quad \text{and} \quad \Gamma(v_{42}) = \text{Commit}(v_{41}, h)$$

then

$$|\Gamma(h)|^{-1}$$

is the LF term representing v42

Commit

```
$ ./gasp commit term.elf
```

if

$$t = \Gamma \vdash v_{52} : \text{Version} \quad \text{and} \quad |\text{term.elf}|_\Gamma = \Delta \vdash h_t : h_A$$

then

$$\Delta [v_{53} = \text{Commit } [prev = v_{52}] [this = h_t]] \vdash v_{53} : \text{Version}$$

is the new repository

Further work

- ▶ still some technical & metatheoretical unknowns
- ▶ from derivations to terms (proof search? views?)
- ▶ diff on terms or derivations
- ▶ type errors handling and recovery
- ▶ mimick other operations from VCS (**Merge**)