Towards typed repositories of proofs

MIPS 2010

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How are *constructed* formal mathematics?

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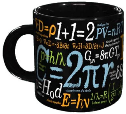
A : They both spend more time *editing* than *writing*



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Observation

We have powerful tools to mechanize the metatheory of (proof) languages

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Isn't it time to make these tools metatheory-aware?

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Rigidity of linear edition

- ((edit; compile)*; commit)* loop does not scale to proofs
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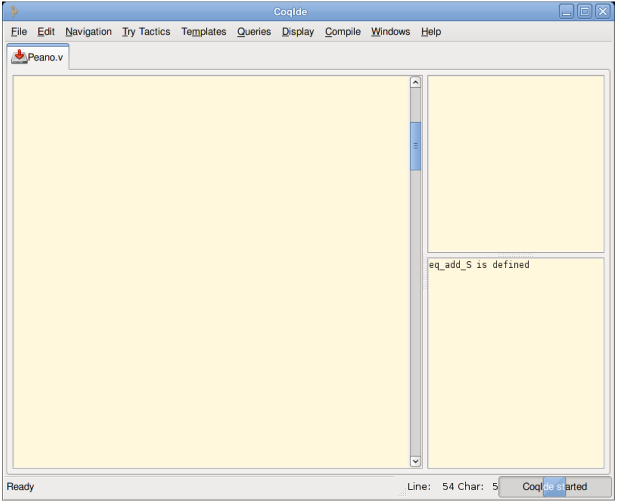
. . . Maybe it wasn't adapted to software development

The impact of changes



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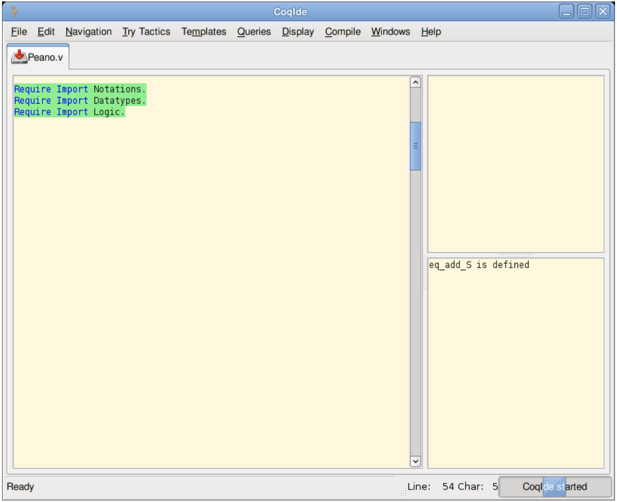

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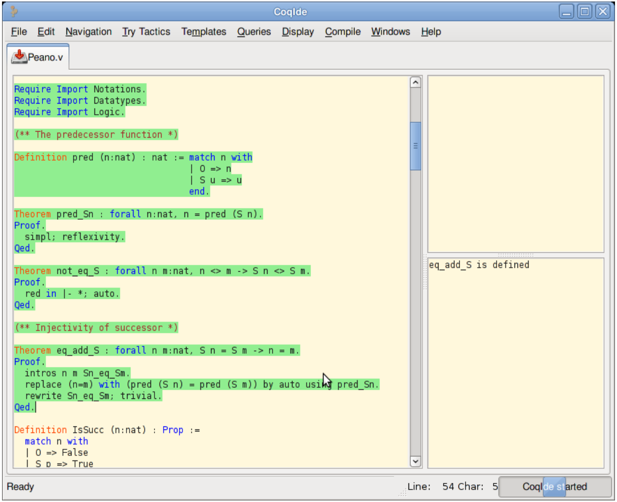

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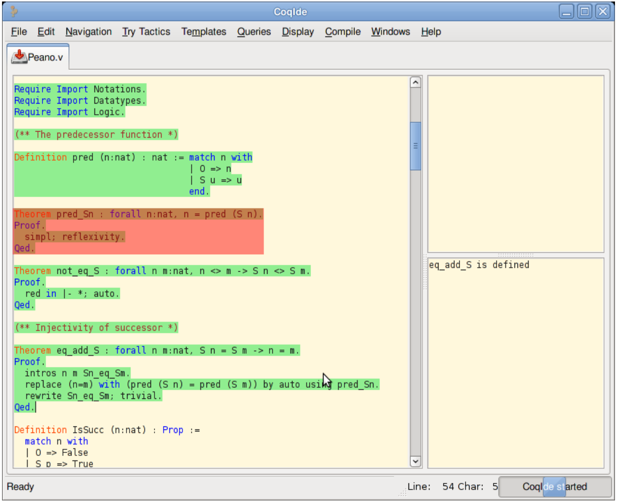

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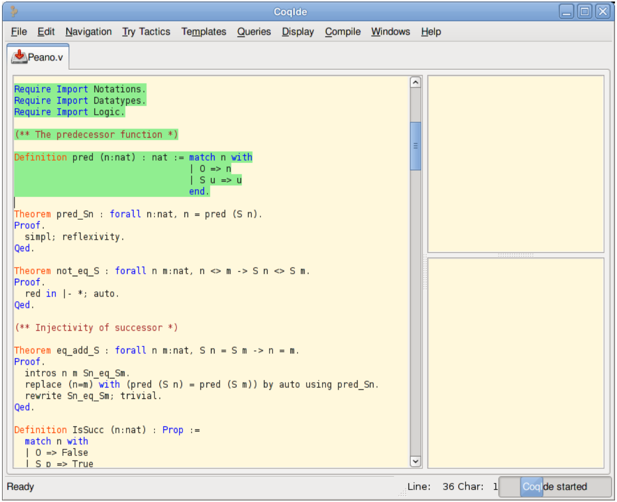

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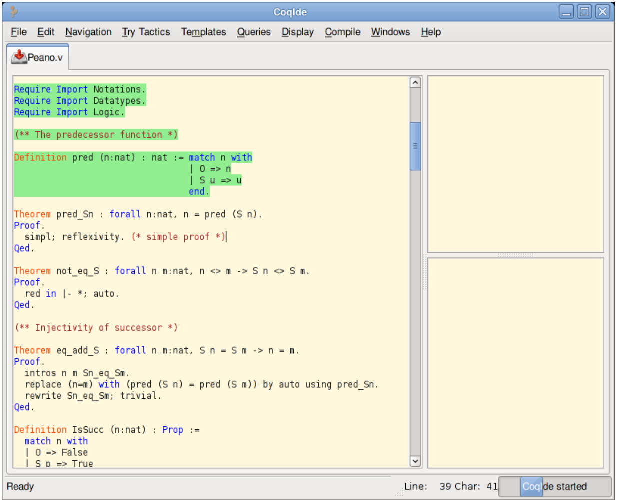

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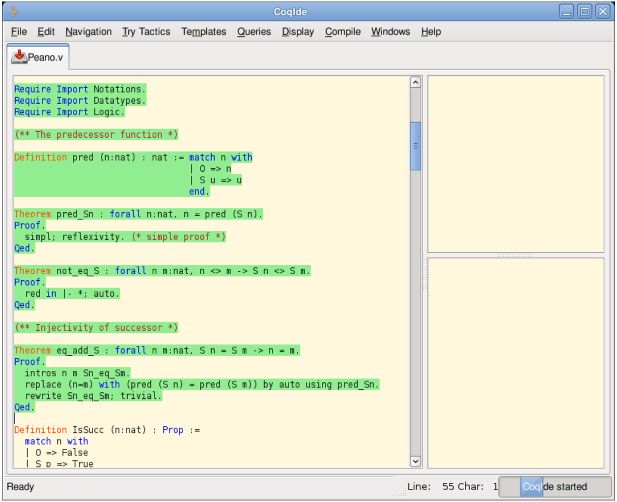

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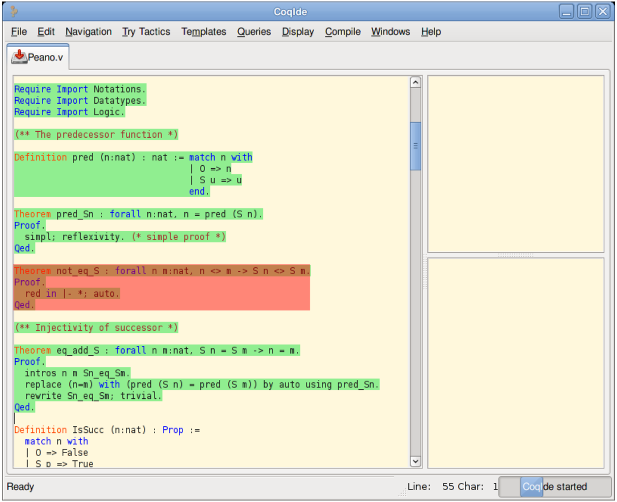

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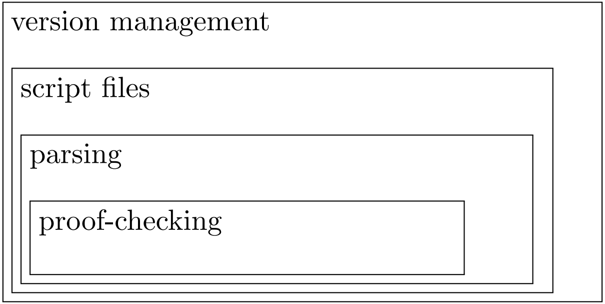

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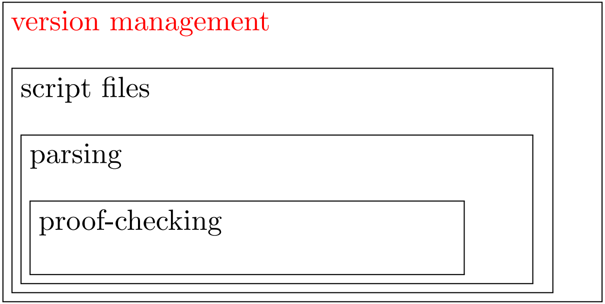

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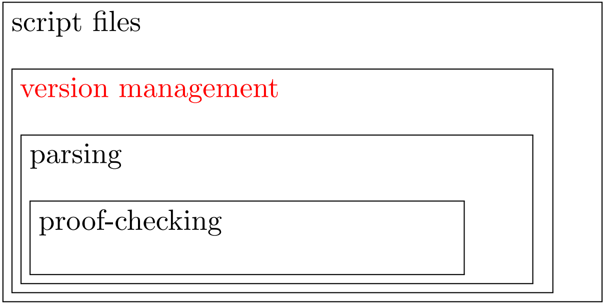

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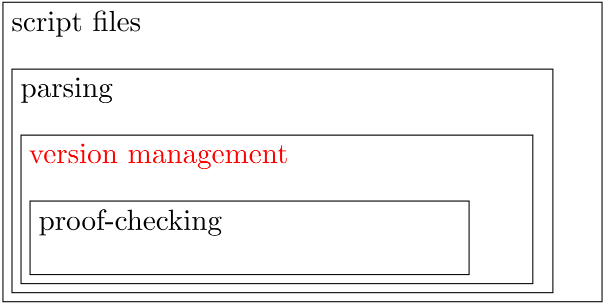

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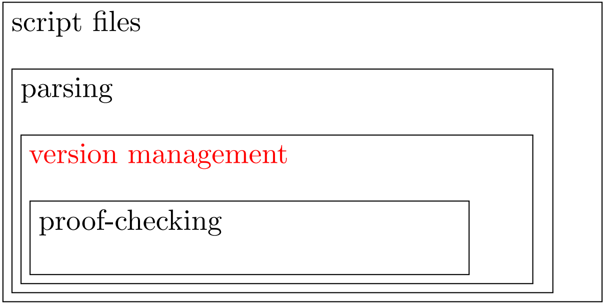

- AST representation
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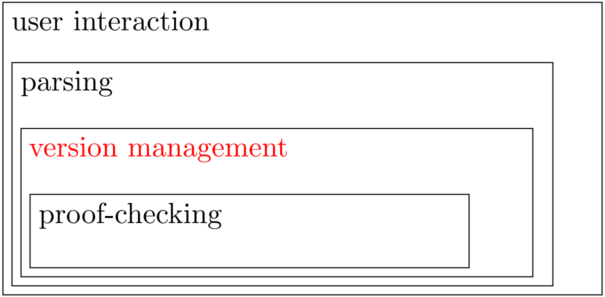

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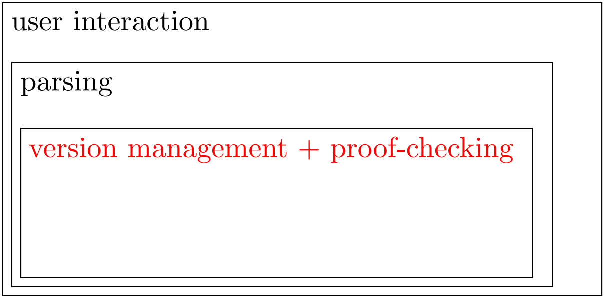

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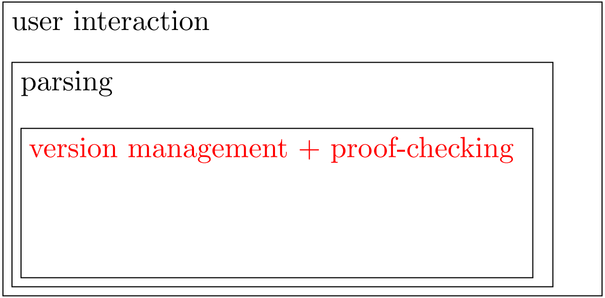

- AST representation
- Explicit dependency DAG
- Typing annotations
- Incremental type-checking

A core meta-language for incremental type-checking

Expresses

- (abstract) Syntax
- (object-) Logics
- Proofs (-terms)

Features

- Typing
- Incrementality
- Dependency

A kernel for a typed version control system?

A repository of directories

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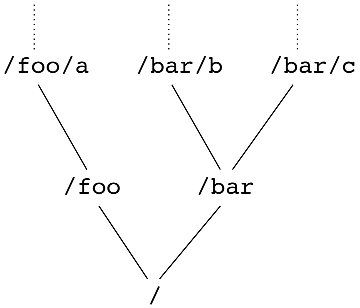

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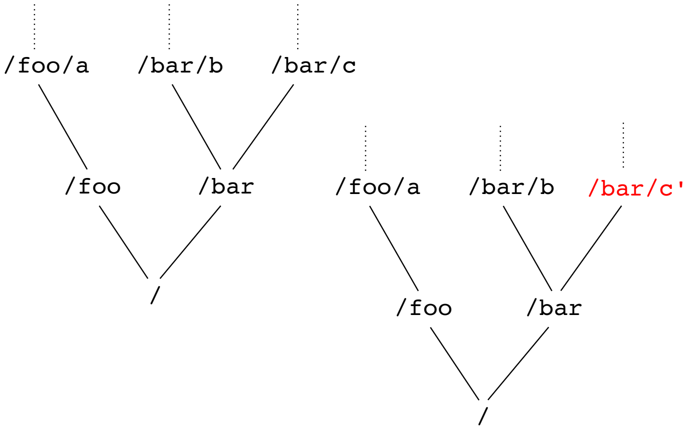

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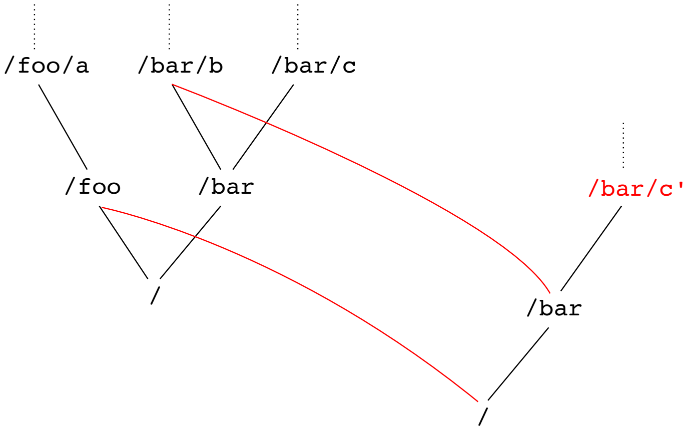

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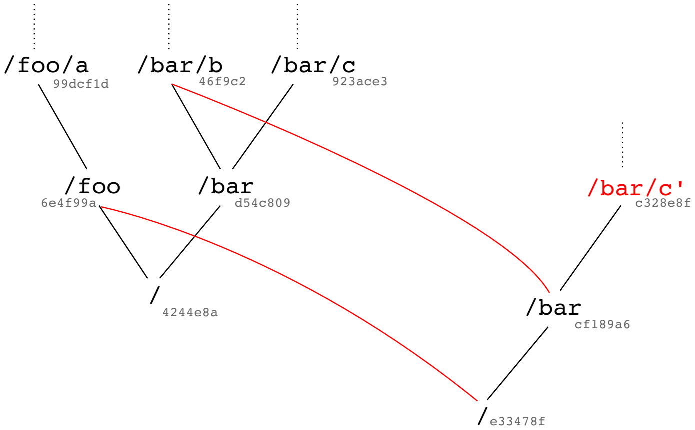

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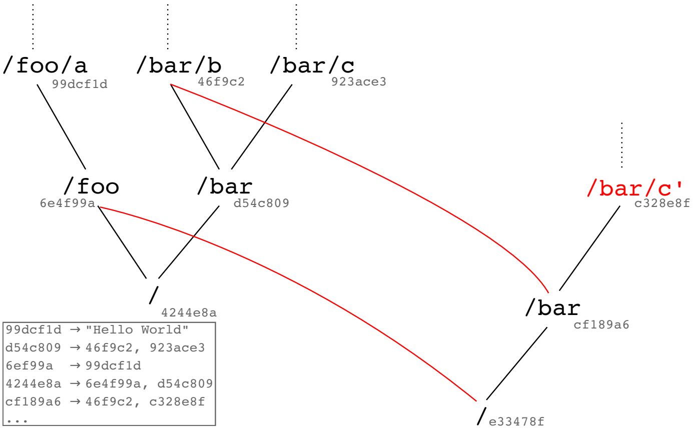

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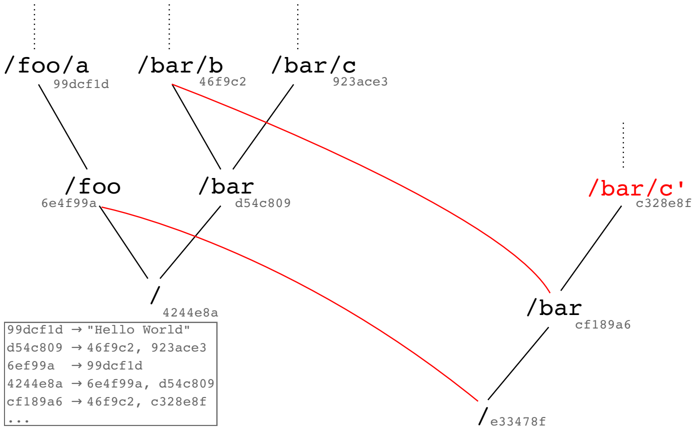

- “Content-adressable”

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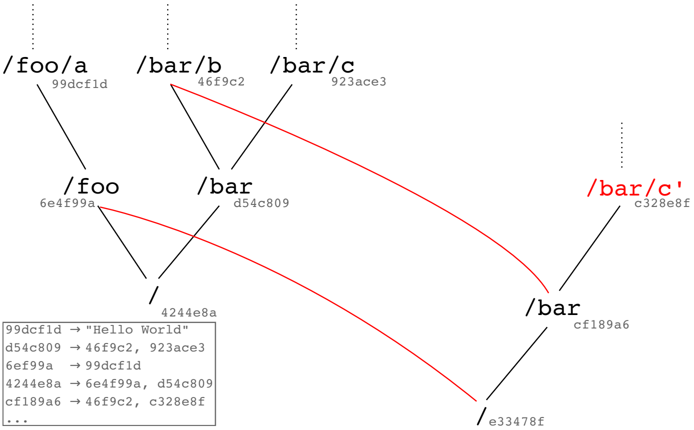

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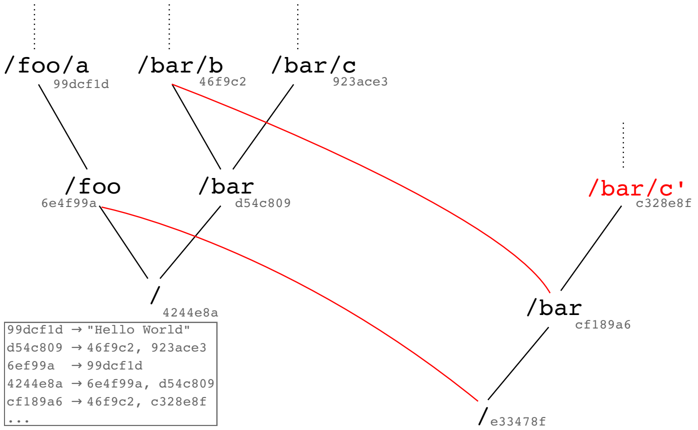

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- Maximal sharing (or hash-consing)

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Let's do the same with *proofs*

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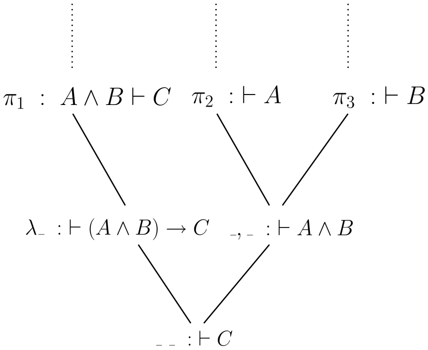
A typed repository of proofs



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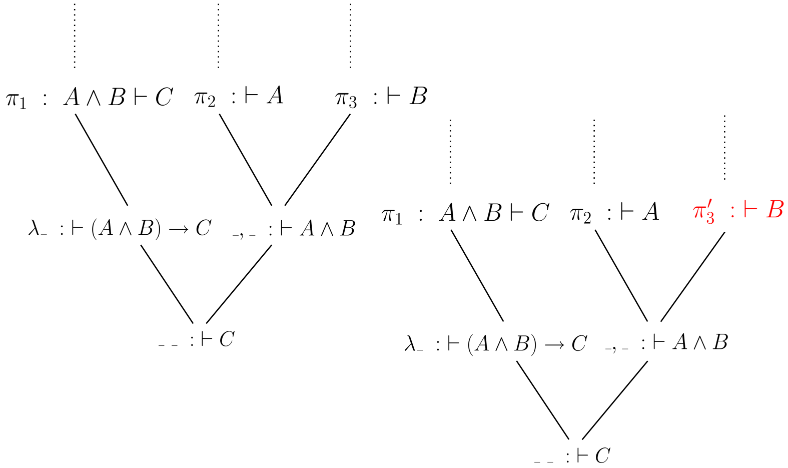

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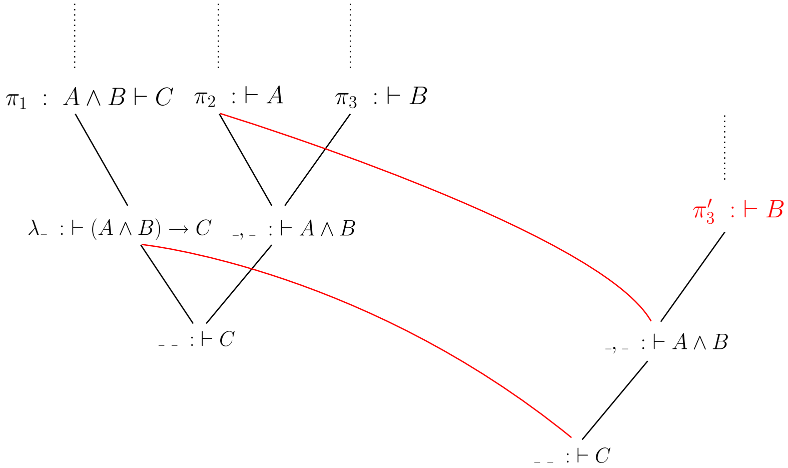

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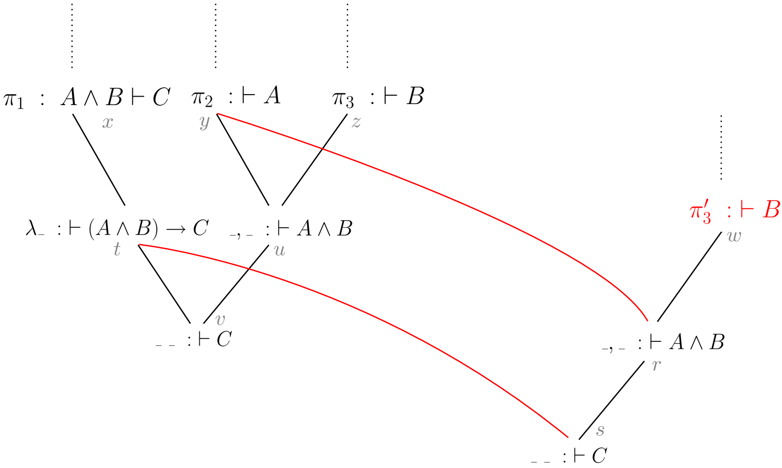

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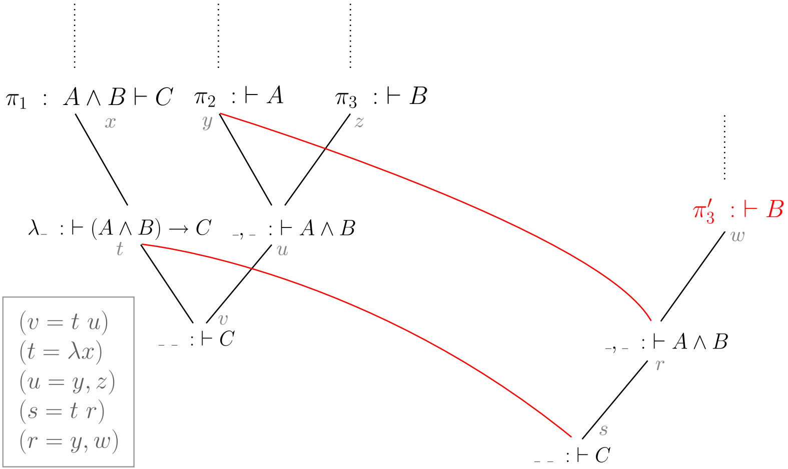

Incremental type-checking, incrementally

Syntax

$$t \;::=\; [x : t] \cdot t \mid (x : t) \cdot t \mid x \mid t\ t \mid *$$

Environments

$$\Gamma \;::=\; \cdot \mid \Gamma[x : t]$$

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$\Gamma \vdash t : u$ " In environment Γ, term t has type u "

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$$\Gamma \vdash t : u \Rightarrow \Delta$$

" From repository Γ, term t of type u leads to the new repository Δ"

Typing rules

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$$\frac{\Gamma \vdash t : * \qquad \Gamma[x : t] \vdash u : *}{\Gamma \vdash (x : t) \cdot u : *}$$

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Application

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Further Work

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1. Constructive metatheory

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1. Constructive metatheory
2. A language to express patches?