A logical framework for incremental type-checking

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May 2011

CEA LIST

A paradoxical situation

Observation

We have powerful tools to mechanize the metatheory of (proof) languages

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Workflow of programming and formal mathematics is still largely inspired by legacy software development (emacs, make, svn, diffs. . .)

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We have powerful tools to mechanize the metatheory of (proof) languages

... And yet,

Workflow of programming and formal mathematics is still largely inspired by legacy software development (emacs, make, svn, diffs...)

Isn't it time to make these tools metatheory-aware?

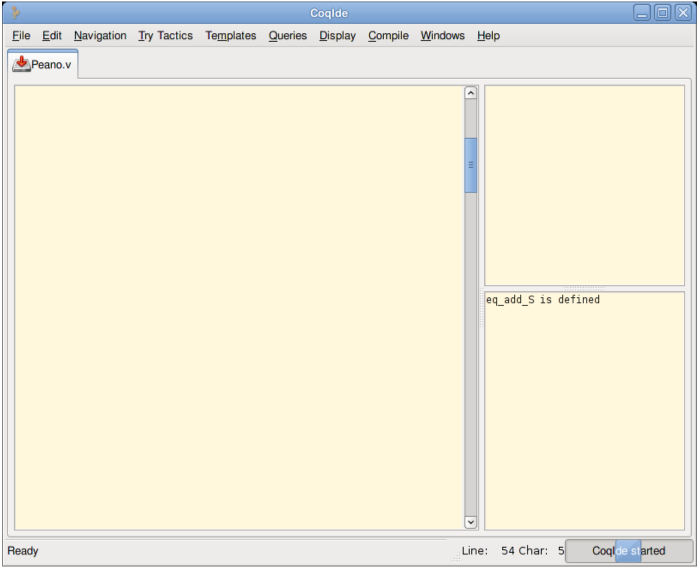
Incrementality in programming & proof languages

Q : Do you spend more time *writing* code or *editing* code?

Today, we use:

- separate compilation
- dependency management
- version control on the scripts
- interactive toplevel with rollback (Coq)

Incrementality in programming & proof languages



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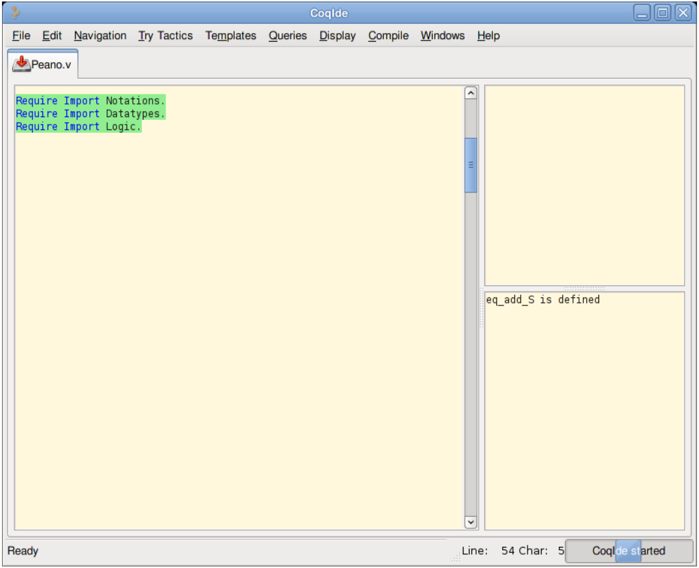

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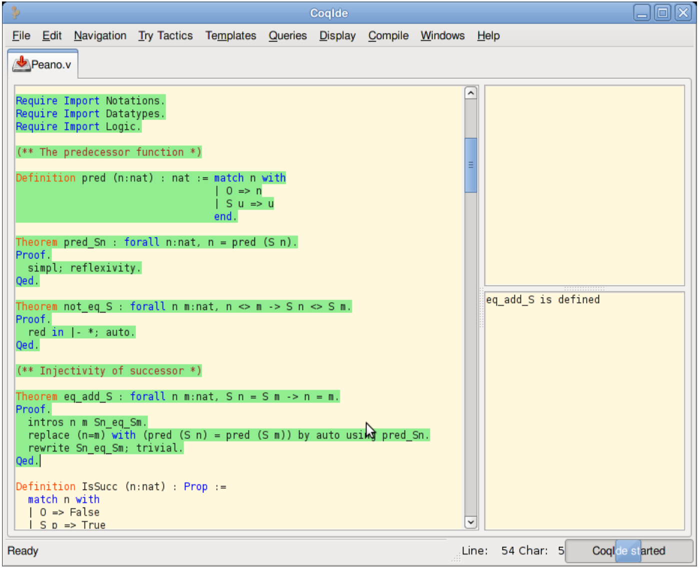

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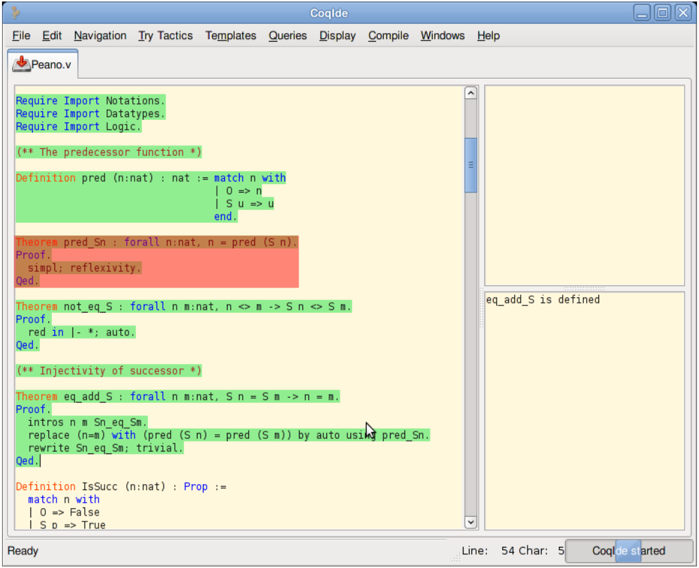

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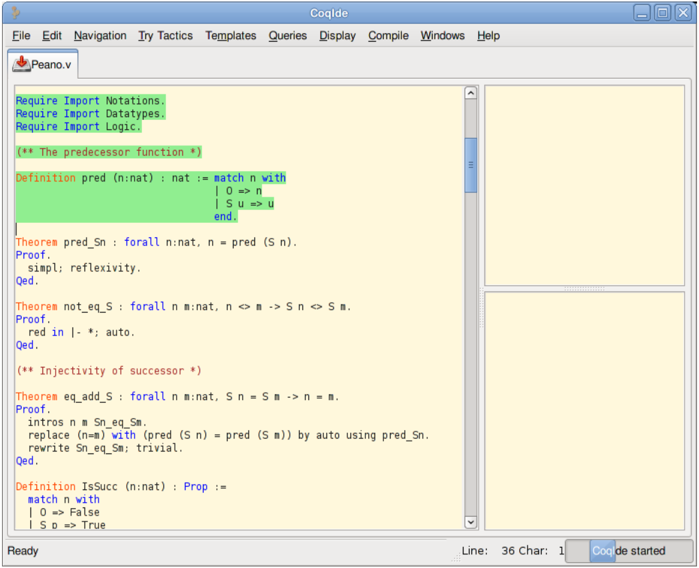

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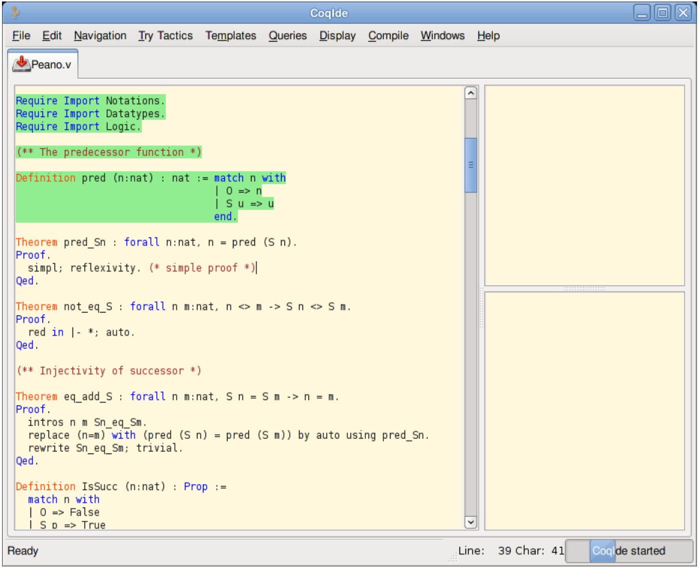

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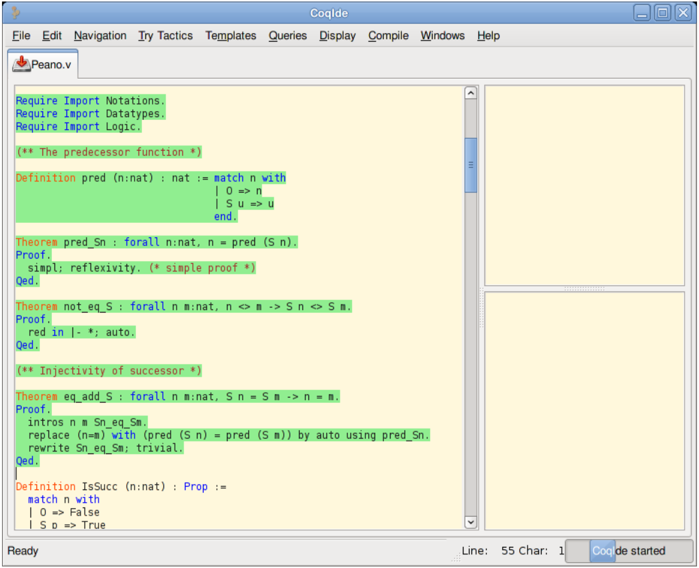

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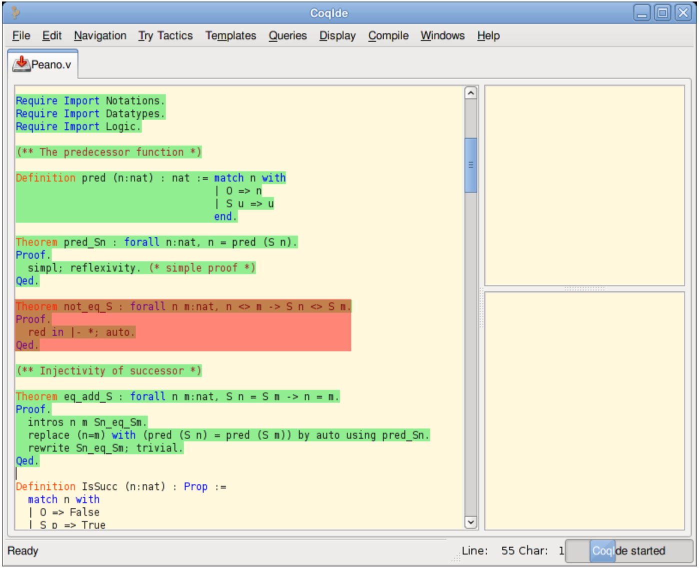

In an ideal world…

- Edition should be possible anywhere
- The impact of changes visible “in real time”
- No need for separate compilation, dependency management

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Applications

- non-(linear|batch) user interaction
- typed version control systems
- type-directed programming
- tactic languages

In this talk, we focus on…

… building a procedure to type-check *local changes*

- What data structure for storing type derivations?
- What language for expressing changes?

Menu

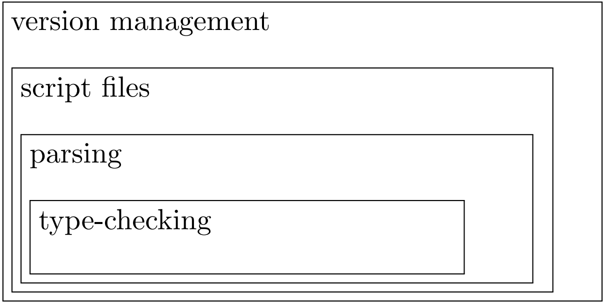
The big picture
- Incremental type-checking
- Why not memoization?

Our approach
- Two-passes type-checking
- The data-oriented way

A metalanguage of repository
- The LF logical framework
- Monadic LF
- Committing to MLF

Menu

The big picture



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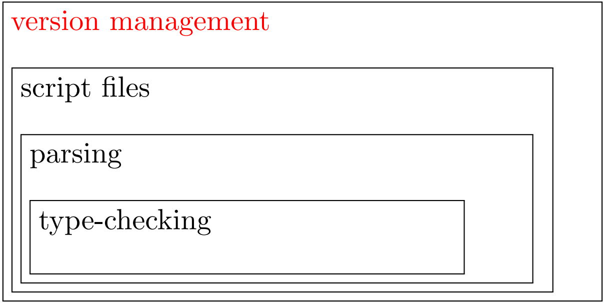

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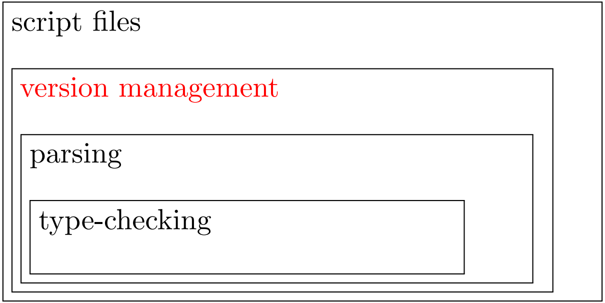

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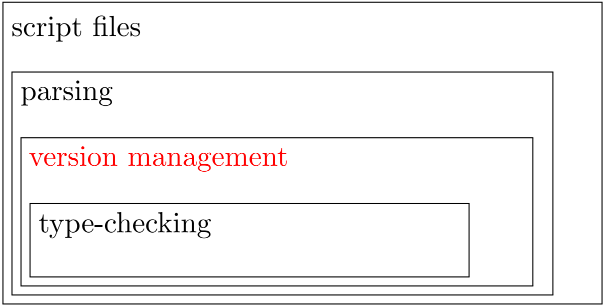
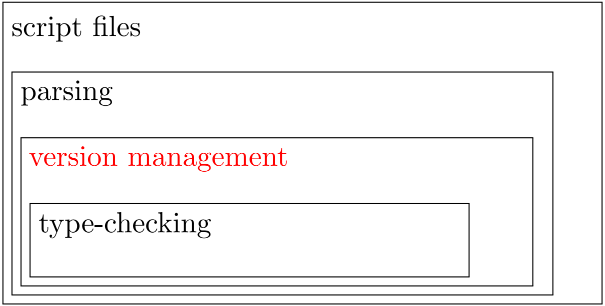


- AST representation

The big picture



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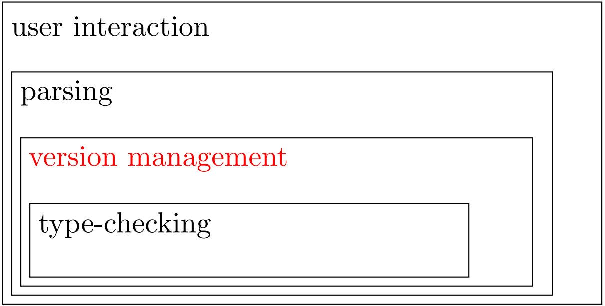

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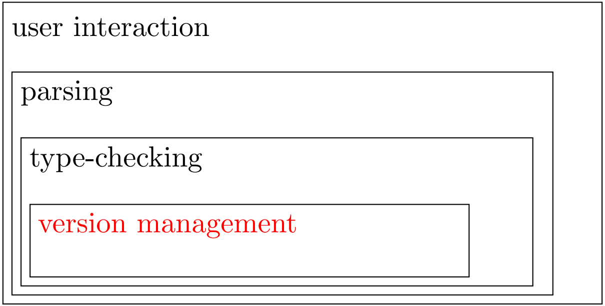

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- Typing annotations

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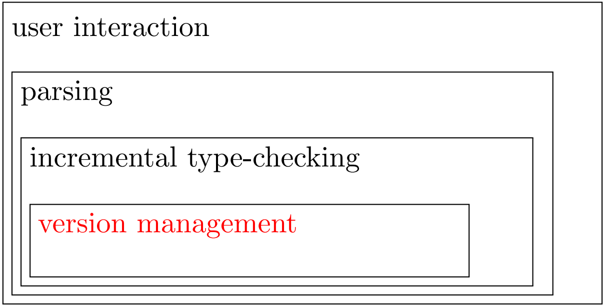

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A logical framework for incremental type-checking

Yes, we're speaking about (any) typed language.

A type-checker

$$\textbf{val}\ \mathsf{check} : \mathsf{env} \to \mathsf{term} \to \mathsf{types} \to \mathsf{bool}$$

- builds and checks the derivation (on the stack)
- conscientiously discards it

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true

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Goal Type-check a large derivation taking advantage of the knowledge from type-checking previous versions

Idea Remember all derivations!

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More precisely

Given a well-typed $\mathcal{R} : \mathit{repository}$ and a $\delta : \mathit{delta}$ and

$$\mathsf{apply} : \mathit{repository} \to \mathit{delta} \to \mathit{derivation} \ ,$$

an incremental type-checker

$$\mathsf{tc} : \mathit{repository} \to \mathit{delta} \to \mathit{bool}$$

decides if $\mathsf{apply}(\delta, \mathcal{R})$ is well-typed in $O(|\delta|)$.
(and not $O(|\mathsf{apply}(\delta, \mathcal{R})|)$)

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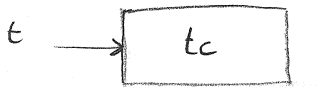

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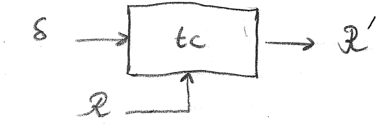

Memoization maybe?

```
let rec check env t a =
  match t with
  | ... → ... false
  | ... → ... true

and infer env t =
  match t with
  | ... → ... None
  | ... → ... Some a
```

Memoization maybe?

```
let table = ref ([] : environ × term × types) in
let rec check env t a =
  if List.mem (env,t,a) !table then true else
    match t with
    | ... → ... false
    | ... → ... table := (env,t,a)::!table; true
and infer env t =
  try List.assoc (env,t) !table with Not_found →
    match t with
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    | ... → ... table := (env,t,a)::!table; Some a
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Syntactically

+ lightweight, efficient implementation

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  What if I want *e.g.* weakening or permutation to be taken into account?

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Semantically

- − external to the type system (meta-cut)
  What does it mean logically?

$$\frac{J \in \Gamma}{\Gamma \vdash J \ \mathsf{wf} \Rightarrow \Gamma} \qquad \frac{\Gamma_1 \vdash J_1 \ \mathsf{wf} \Rightarrow \Gamma_2 \qquad \dots \qquad \Gamma_{n-1}[J_{n-1}] \vdash J_n \ \mathsf{wf} \Rightarrow \Gamma_n}{\Gamma_1 \vdash J \ \mathsf{wf} \Rightarrow \Gamma_n[J_n][J]}$$

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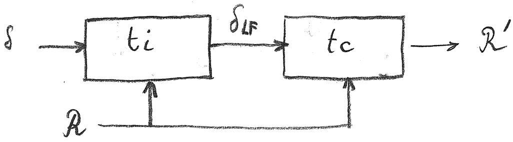
Mixes two goals: *derivation synthesis* & *object reuse*

Menu

Two-passes type-checking



$$\begin{aligned}
\text{ti} &= \text{type inference} = \text{derivation delta synthesis} \\
\text{tc} &= \text{type checking} = \text{derivation delta checking} \\
\delta &= \text{program delta} \\
\delta_{LF} &= \text{derivation delta} \\
\mathcal{R} &= \text{repository of derivations}
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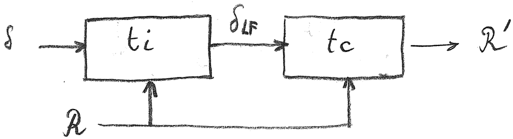

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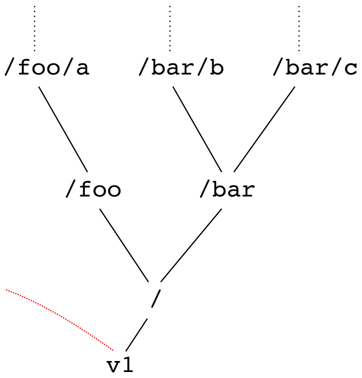
Shift of trust: ti (complex, ad-hoc algorithm) $\rightarrow$ tc (simple, generic kernel)

A popular storage model for directories



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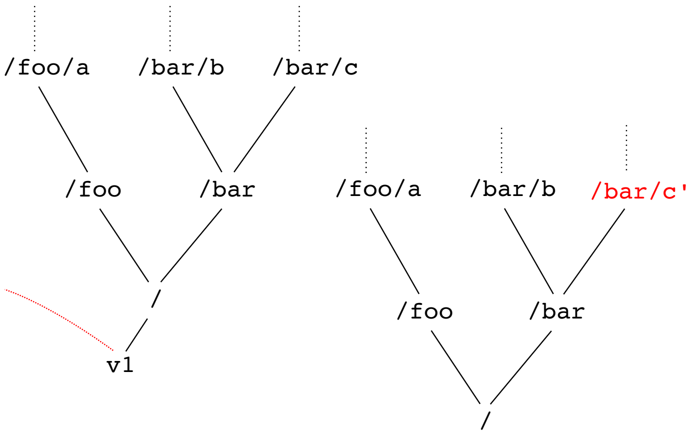

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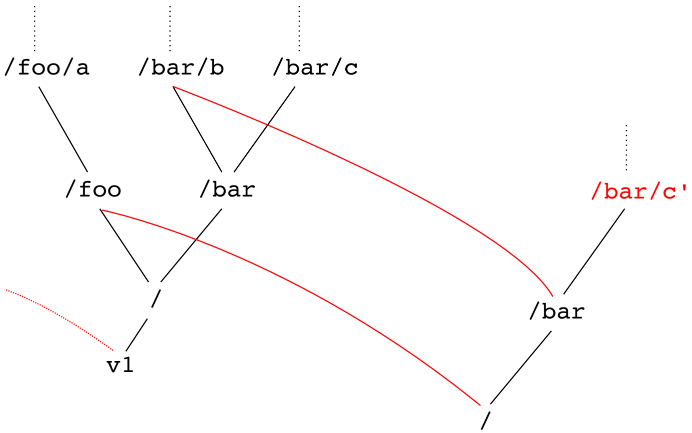

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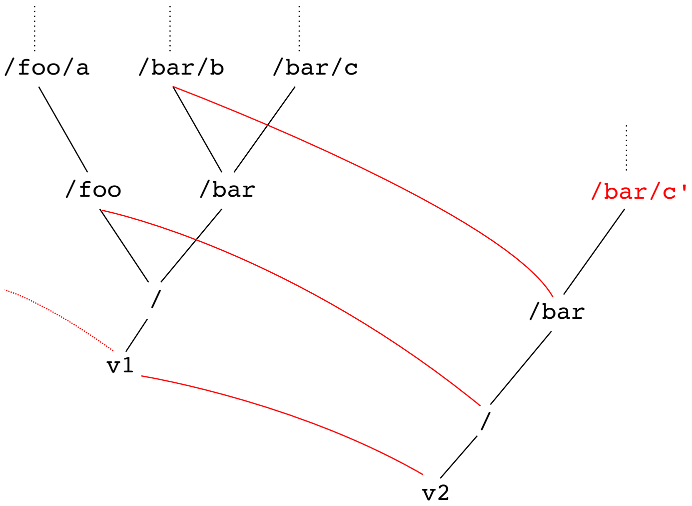

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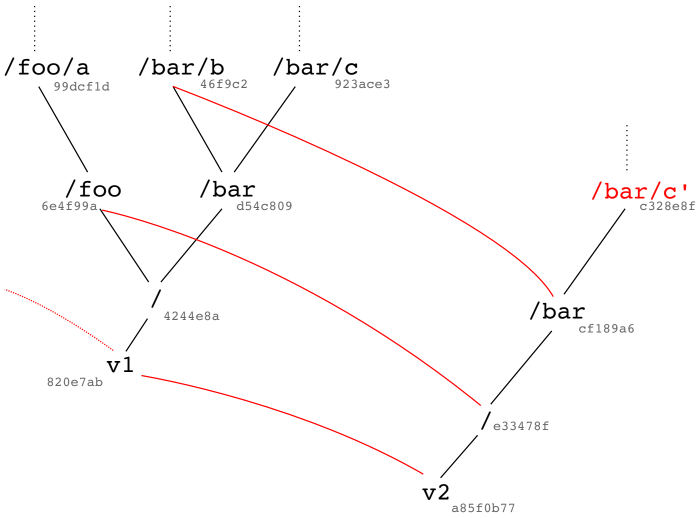

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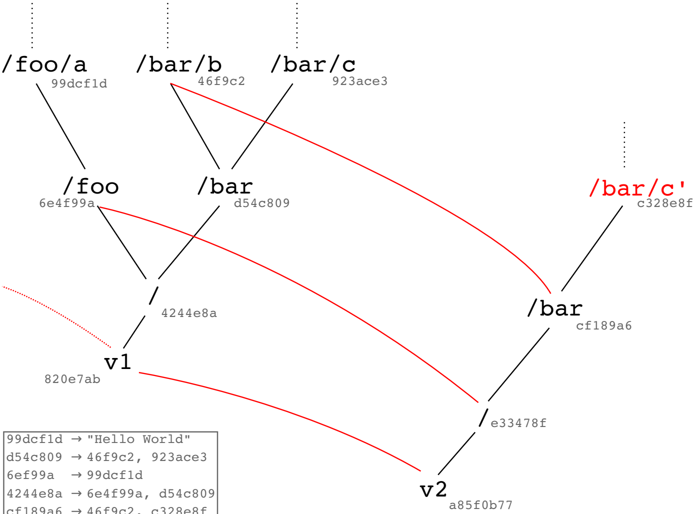


```
99dcf1d  → "Hello World"
d54c809  → 46f9c2, 923ace3
6ef99a   → 99dcf1d
4244e8a  → 6e4f99a, d54c809
cf189a6  → 46f9c2, c328e8f
...
```

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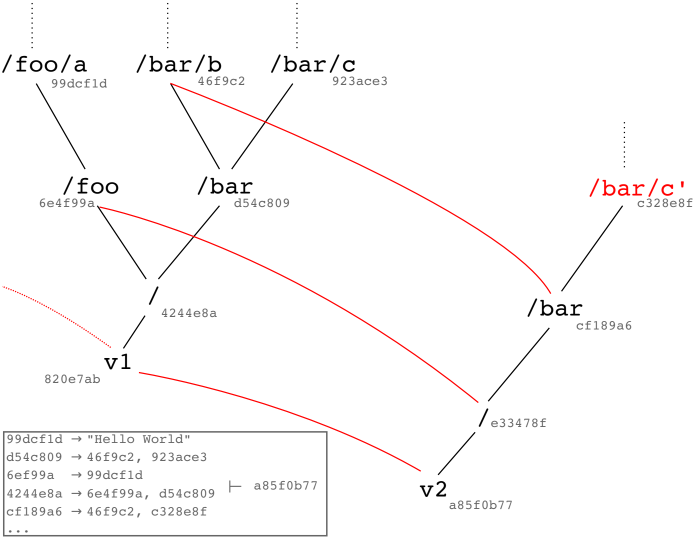

A popular storage model for directories

The repository $\mathcal{R}$ is a pair (Δ, x):

$$\Delta : x \mapsto (\mathsf{Commit}\ (x \times y) \mid \mathsf{Tree}\ \vec{x} \mid \mathsf{Blob}\ string)$$

Operations

`commit` δ
- extend the database with Tree/Blob objects
- add a Commit object
- update head

`checkout` v
- follow v all the way to the Blobs

`diff` v_1 v_2
- follow simultaneously v_1 and v_2
- if object names are equal, stop (content is equal)
- otherwise continue

...

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- Δ forms a DAG
- if $(x, \mathsf{Commit}\ (y, z)) \in \Delta$ then
  - $(y, \mathsf{Tree}\ t) \in \Delta$
  - $(z, \mathsf{Commit}\ (t, v)) \in \Delta$
- if $(x, \mathsf{Tree}(\vec{y})) \in \Delta$ then
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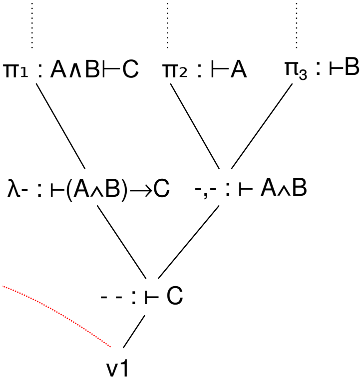
Let's do the same with *proofs*

A *typed* repository of proofs



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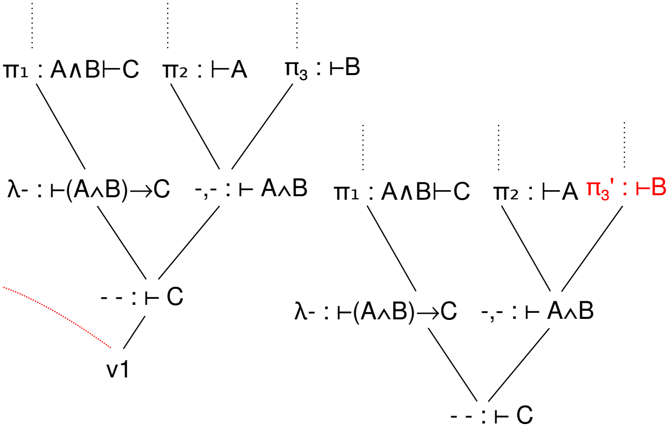

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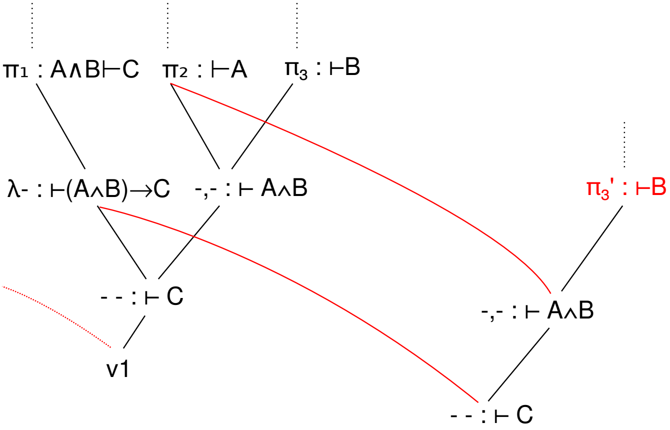

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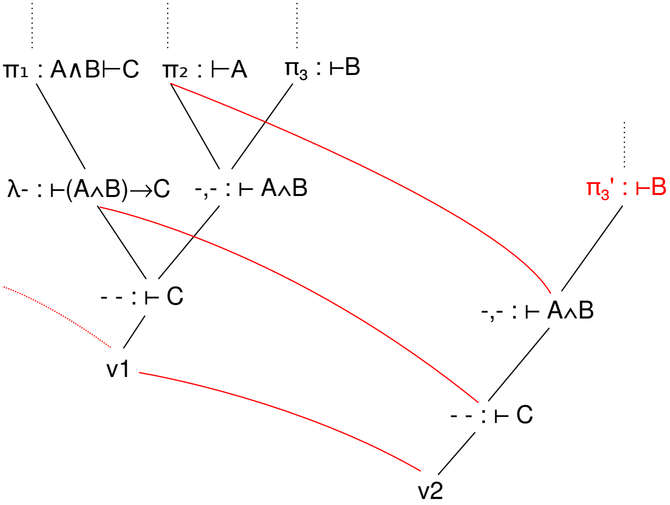

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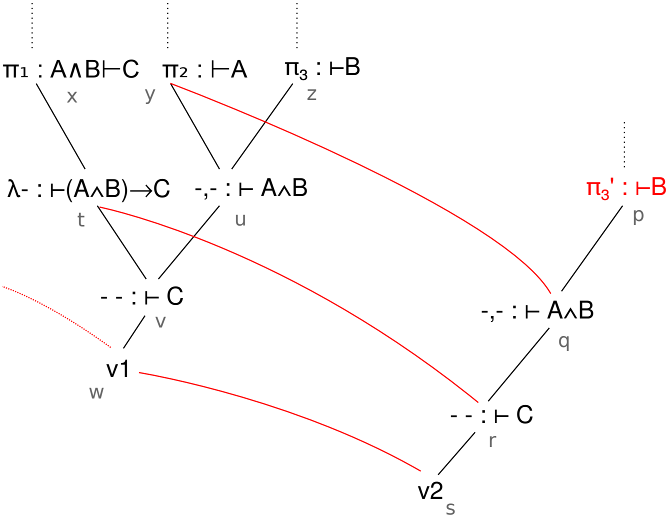

A *typed* repository of proofs

$$
\begin{aligned}
x &= \ldots \;: A \wedge B \vdash C \\
y &= \ldots \;: \vdash A \\
z &= \ldots \;: \vdash B \\
t &= \lambda a : A \wedge B \cdot x : \vdash A \wedge B \to C \\
u &= (y, z) : \vdash A \wedge B \\
v &= t\ u : \vdash C \\
w &= \textsf{Commit}(v, w1) : \textsf{Version}
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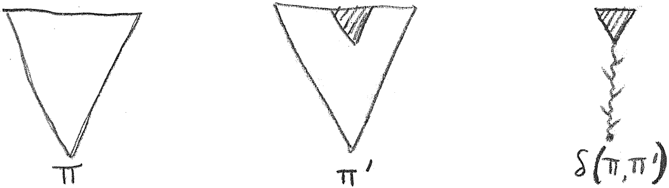
A data-oriented notion of delta

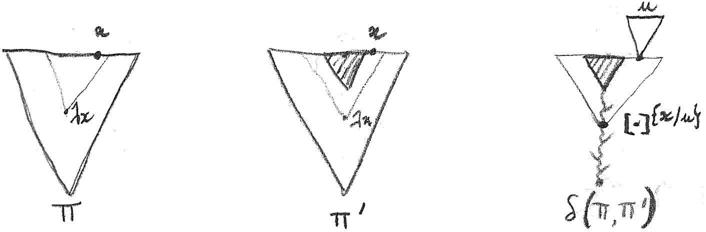
The first-order case



A delta is a term t with *variables* x, y , defined in the repository

A data-oriented notion of delta

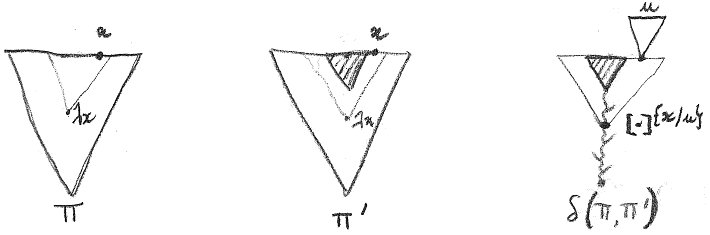
The binder case



A delta is a term t with *variables* x, y and *boxes* $[t]^{\{x/u\}}_{y.n}$ to jump over binders in the repository

A data-oriented notion of delta

The binder case



A delta is a term t with *variables* x, y and *boxes* $[t]_{y.n}^{\{x/u\}}$ to jump over binders in the repository

Towards a metalanguage of proof repository

Repository language

1. name all proof steps
2. annote them by judgement

Delta language

1. address sub-proofs (variables)
2. instantiate lambdas (boxes)
3. check against $\mathcal{R}$

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$\leadsto$ **Need extra-logical features!**

Menu

A logical framework for incremental type-checking

LF [Harper et al. 1992] (a.k.a. $\lambda\Pi$) provides a **meta-logic** to represent and validate syntax, rules and proofs of an **object language**, by means of a typed λ-calculus.

dependent types to express object-judgements

signature to encode the object language

higher-order abstract syntax to easily manipulate hypothesis

A logical framework for incremental type-checking

LF [Harper et al. 1992] (a.k.a. $\lambda\Pi$) provides a **meta-logic** to represent and validate syntax, rules and proofs of an **object language**, by means of a typed λ-calculus.

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Examples

- $$\dfrac{\begin{array}{c}[x : A] \\ \vdots \\ t : B\end{array}}{\lambda x \cdot t : A \to B} \quad \rightsquigarrow \quad \begin{array}{ll}\mathsf{is\text{-}lam} : & \Pi A, B : \mathsf{ty} \cdot \Pi t : \mathsf{tm} \to \mathsf{tm} \cdot \\ & (\Pi x : \mathsf{tm} \cdot \mathsf{is}\ x\ A \to \mathsf{is}\ (t\ x)\ B) \to \\ & \mathsf{is}\ (\mathsf{lam}\ A\ (\lambda x \cdot t\ x))(\mathsf{arr}\ A\ B)\end{array}$$
- $$\dfrac{[x : \mathbb{N}]}{\lambda x \cdot x : \mathbb{N} \to \mathbb{N}} \quad \rightsquigarrow \quad \begin{array}{l}\mathsf{is\text{-}lam}\ \mathsf{nat}\ \mathsf{nat}\ (\lambda x \cdot x)\ (\lambda yz \cdot z) \\ \quad : \mathsf{is}\ (\mathsf{lam}\ \mathsf{nat}\ (\lambda x \cdot x))\ (\mathsf{arr}\ \mathsf{nat}\ \mathsf{nat})\end{array}$$

A logical framework for incremental type-checking

Syntax

$$\begin{array}{rcl} K & ::= & \Pi x : A \cdot K \mid * \\ A & ::= & \Pi x : A \cdot A \mid a(l) \\ t & ::= & \lambda x : A \cdot t \mid x(l) \mid c(l) \\ l & ::= & \cdot \mid t, l \\ \Sigma & ::= & \cdot \mid \Sigma[c : A] \mid \Sigma[a : K] \end{array}$$

Judgements

- $\Gamma \vdash_\Sigma K$
- $\Gamma \vdash_\Sigma A : K$
- $\Gamma \vdash_\Sigma t : A$
- $\vdash \Sigma$

The delta language

Syntax

$$K ::= \Pi x : A \cdot K \mid *$$
$$A ::= \Pi x : A \cdot A \mid a(l)$$
$$t ::= \lambda x : A \cdot t \mid x(l) \mid c(l) \mid [t]^{\{x/t\}}_{x.n}$$
$$l ::= \cdot \mid t, l$$
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Judgements

- $\mathcal{R}, \Gamma \vdash_\Sigma K \Rightarrow \mathcal{R}$
- $\mathcal{R}, \Gamma \vdash_\Sigma A : K \Rightarrow \mathcal{R}$
- $\mathcal{R}, \Gamma \vdash_\Sigma t : A \Rightarrow \mathcal{R}$
- $\vdash \Sigma$

Informally

- $\mathcal{R}, \Gamma \vdash_\Sigma x \Rightarrow \mathcal{R}$ means
  "I am what x stands for, in Γ or in $\mathcal{R}$ (and produce $\mathcal{R}$)".
- $\mathcal{R}, \Gamma \vdash_\Sigma [t]^{\{x/u\}}_{y.n} \Rightarrow \mathcal{R}'$ means
  "Variable y has the form $\_(v_1 \ldots v_{n-1}(\lambda x \cdot \mathcal{R}'') \ldots)$ in $\mathcal{R}$. Make all variables in $\mathcal{R}''$ in scope for t, taking u for x. t will produce $\mathcal{R}'$"

Naming of proof steps

Remark

In LF, proof step = term application spine

Example $\mathsf{is\text{-}lam\ nat\ nat}\ (\lambda x \cdot x)\ (\lambda yz \cdot z)$

Monadic Normal Form (MNF)

Program transformation, IR for FP compilers

Goal: sequentialize all computations by naming them (lets)

$$\begin{array}{lcl} t & ::= & \lambda x \cdot t \mid t(l) \mid x \\ l & ::= & \cdot \mid t, l \end{array} \quad \Longrightarrow \quad \begin{array}{lcl} \underline{t} & ::= & \mathsf{ret}\ \underline{v} \mid \mathsf{let}\ x = \underline{v}(\underline{l})\ \mathsf{in}\ \underline{t} \mid \underline{v}(\underline{l}) \\ \underline{l} & ::= & \cdot \mid \underline{v}, \underline{l} \\ \underline{v} & ::= & x \mid \lambda x \cdot \underline{t} \end{array}$$

Examples

- $f(g(x)) \quad \notin \quad \mathsf{MNF}$
- $\lambda x \cdot f(g(\lambda y \cdot y, x)) \quad \Longrightarrow$
  $\mathsf{ret}\ (\lambda x \cdot \mathsf{let}\ a = g(\lambda y \cdot y, x)\ \mathsf{in}\ f(a))$

Naming of proof steps

Positionality inefficiency

$$\begin{array}{l} \mathsf{let}\ x = \ldots\ \mathsf{in} \\ \quad \mathsf{let}\ y = \ldots\ \mathsf{in} \\ \quad\quad \mathsf{let}\ z = \ldots\ \mathsf{in} \\ \quad\quad\quad \vdots \\ \quad\quad\quad\quad \underline{v}(\underline{l}) \end{array}$$

Naming of proof steps

Positionality inefficiency

$$
\begin{array}{l}
\mathsf{let}\ x = \ldots\ \mathsf{in} \\
\quad \mathsf{let}\ y = \ldots\ \mathsf{in} \\
\qquad \mathsf{let}\ z = \ldots\ \mathsf{in} \\
\qquad\quad \vdots \\
\qquad\qquad \underline{v}(\underline{l})
\end{array}
\quad\Longrightarrow\quad
\left(\begin{array}{l}
x = \ldots \\
y = \ldots \\
z = \ldots \\
\vdots
\end{array}\right) \vdash \underline{v}(\underline{l})
$$

Naming of proof steps

Positionality inefficiency

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Non-positional monadic calculus

$$\begin{array}{rcl}\underline{t} & ::= & \mathsf{ret}\ \underline{v} \mid \mathsf{let}\ x = \underline{v}(\underline{l})\ \mathsf{in}\ \underline{t} \mid \underline{v}(\underline{l}) \\ \underline{l} & ::= & \cdot \mid \underline{v}, \underline{l} \\ \underline{v} & ::= & x \mid \lambda x \cdot \underline{t}\end{array}$$

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\quad \Longrightarrow \quad
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Non-positional monadic calculus

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\begin{array}{rcl}
\underline{t} & ::= & \mathsf{ret}\ \underline{v} \mid \underline{\sigma} \vdash \underline{v}(\underline{l}) \\
\underline{l} & ::= & \cdot \mid \underline{v}, \underline{l} \\
\underline{v} & ::= & x \mid \lambda x \cdot \underline{t} \\
\underline{\sigma} & ::= & \cdot \mid \underline{\sigma}[x = \underline{v}(\underline{l})]
\end{array}
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Monadic LF

$$
\begin{array}{rcl}
K & ::= & \Pi x : A \cdot K \mid * \\
A & ::= & \Pi x : A \cdot A \mid \sigma \vdash a(l) \\
t & ::= & \mathsf{ret}\ v \mid \sigma \vdash h(l) \\
h & ::= & x \mid c \\
l & ::= & \cdot \mid v, l \\
v & ::= & c \mid x \mid \lambda x : A \cdot t \\
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Type annotation

Remark

In LF, judgement annotation = type annotation

Example

is-lam nat nat $(\lambda x \cdot x)$ $(\lambda yz \cdot z)$

: is (lam nat $(\lambda x \cdot x)$) (arr nat nat)

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The repository language

Remark

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is-lam nat nat $(\lambda x \cdot x)$ $(\lambda yz \cdot z)$
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$$
\begin{array}{rcl}
K & ::= & \Pi x : A \cdot K \mid * \\
A & ::= & \Pi x : A \cdot A \mid \sigma \vdash a(l) \\
\mathcal{R} & ::= & \sigma \vdash h(l) : a(l) \\
h & ::= & x \mid a \\
l & ::= & \cdot \mid v, l \\
v & ::= & c \mid x \mid \lambda x : A \cdot \mathcal{R} \\
\sigma & : & x \mapsto h(l) : a(l) \\
\Sigma & ::= & \cdot \mid \Sigma[c : A] \mid \Sigma[a : K]
\end{array}
$$

Commit (WIP)

$$\mathcal{R}^-, \cdot^- \vdash_{\Sigma^-} t^- : A^+ \Rightarrow \mathcal{R}^+$$

What does it do?

- type-checks t *wrt.* $\mathcal{R}$ (in $O(t)$)
- puts t in non-pos. MNF
- annotate with type
- with the adapted rules for variable & box:

$$\textsc{Var}\ \frac{\Gamma(x) = A \qquad \text{or} \qquad \sigma(x) : A}{(\sigma \vdash \_ : \_), \Gamma \vdash_\Sigma x : A \Rightarrow (\sigma \vdash x : A)}$$

$$\textsc{Box}\ \frac{\sigma(x).i = \lambda y : B \cdot (\rho \vdash H'') \qquad (\sigma \vdash H), \Gamma \vdash u : B \Rightarrow (\theta \vdash H') \qquad (\rho \cup \theta[y = H'] \vdash H''), \Gamma \vdash t : A \Rightarrow \mathcal{R}}{(\sigma \vdash H), \Gamma \vdash [t]_{x.i}^{\{y/u\}} : A \Rightarrow \mathcal{R}}$$

Example

Signature

$$A\ B\ C\ D : *$$

$$a : (D \to B) \to C \to A \qquad\qquad b\ b' : C \to B$$

$$c : D \to C \qquad\qquad d : D$$

Terms

$$\begin{aligned}
t_1 &= a(\lambda x : D \cdot b(c(x)), c(d)) \\
\mathcal{R}_1 &= [v = c(d) : C] \vdash a(\lambda x : D \cdot [w = c(x) : C] \vdash b(w) : B, v) : A \\
t_2 &= a(\lambda y : D \cdot [b'(w)]_1^{\{x/y\}}) \\
\mathcal{R}_2 &= [v = c(d) : C] \vdash \\
&\qquad a(\lambda y : D \cdot [x = y][w = c(x) : C] \vdash b'(w) : B, v) : A
\end{aligned}$$

Regaining version management

Just add to the signature Σ:

$$\begin{aligned} \mathsf{Version} &: * \\ \mathsf{Commit0} &: \mathsf{Version} \\ \mathsf{Commit} &: \Pi t : \mathsf{tm} \cdot \mathsf{is}(t, \mathsf{unit}) \to \mathsf{Version} \to \mathsf{Version} \end{aligned}$$

Commit t

if $\quad \mathcal{R} = \sigma \vdash v : \mathsf{Version} \quad$ and $\quad \mathcal{R}, \cdot \vdash_\Sigma t : \mathsf{is}(p, \mathsf{unit}) \Rightarrow (\rho \vdash k)$

then

$$\rho[x = \mathsf{Commit}(p, k, v)] \vdash x : \mathsf{Version}$$

is the new repository

Further work

- implementation & metatheory of Commit
- from terms to derivations (ti)
- diff on terms
- mimick other operations from VCS (Merge)